Schemes for generating W state of paths and W state of polarization photons

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Entanglement is the key physical resource in most quantum information process, e. g., quantum teleportation [1], quantum key distribution [2], quantum computation[3] and so on. In multiparticle case, it was shown that there exists two inequivalent classes of entangled states, namely Greenberger-Horne-Zeilinger (GHZ) [4] state and W state [5], where, for example in three-particle case, $|GHZ\rangle = \frac{1}{\sqrt{2}}[|000\rangle + |111\rangle]$ and $|W\rangle = \frac{1}{\sqrt{3}}[|001\rangle + |010\rangle + |100\rangle]$, they can not been converted to each other even under stochastic local operations and classical communication. Recently, Cabello considered a set of Bell inequality to show some differences between the violation of local realism exhibited by GHZ state and W state.[6] One obvious application of general W state is telecloning [7, 8]. If we choose the particular W state $|\Psi_{clone}\rangle = \sqrt{\frac{2}{3}}|100\rangle - \sqrt{\frac{1}{6}}|010\rangle - \sqrt{\frac{1}{6}}|001\rangle$, then we can get precisely a Bužek-Hillery cloning [9] from the sender to two receivers, provided that the results are averaged over the four possible measurement outcomes of Bell state by sender. In Ref. [10], we give a scheme by which W state can be used to realize the teleportation of an unknown state probabilisticly. The teleportation of the entangled state and dense coding by W state are discussed in Ref. [11]. In this paper, we consider how to produce a W state. The GHZ state has been produced in laboratory recently using Spontaneous-Parametric-Downcoversion. [12] How about W state? In Ref. [13], authors present a quantum electrodynamics scheme, by which, a multiatom W state can be produced. The Heisenberg model was used to produce three-atom or four-atom W state in Ref. [14]. Very recently, Zou et. al [15] present a scheme by which a four-photon or a three-photon W state can be generated by linear optical elements. In their scheme, besides many linear optical elements, a maximal entangled state source and single photon source are needed. The setup seems complicated. In this paper, we give two very simple schemes, by which, two kinds of W state can be produced very easily. The first kind is the path W state, the character of the first kind W state is that the total number of particle is just one. The successful probability is 100%. The second is the multiphoton polarization W state, we can get it probabilisticly with postselection. In these two schemes, just a common commercial multiport fiber coupler and single photon source are needed. Contrast to the scheme [15], no maximal entangled source and other linear optic elements are needed. So, they are very simple and easy to do in practice. Furthermore, by these schemes, not only a three-mode or a four mode W state (first kind) and a three-photon or a four-photon W state (second kind), but also an arbitrary multimode (first kind) and multiphoton (second kind) W state can be produced in principle, so our schemes are more general than the scheme in Ref. [15]. In the follow, we give these schemes to produce two kinds of different W states respectively. Firstly, we discuss how to generate the first kind of W state.

We take how to produce the state $|W\rangle = \sqrt{\frac{1}{3}}[|001\rangle + |010\rangle + |100\rangle]_{123}$ as an example, where, subscripts 1, 2, 3 refer to the different three space modes, and $|0\rangle$ means vacuum state, $|1\rangle$ means one photon state. In order to do it, what we need is just a 3×3 symmetric fiber coupler (tritter) if we do it in fiber system. A symmetric 3×3 fiber can be described by a unimodulat matrix. A standard form of the tritter matrix T, in which the first column and the first row are real, is given by [16]

$$T = \sqrt{\frac{1}{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \exp(i\frac{2\pi}{3}) & \exp(i\frac{4\pi}{3}) \\ 1 & \exp(i\frac{4\pi}{3}) & \exp(i\frac{2\pi}{3}) \end{bmatrix}. \qquad (1)$$

If the input state to the tritter is $\Psi_{in} = (1, 0, 0)_{123}$, where, 1, 2, 3 refer to three input ports of tritter, one photon is in input 1, the inputs to 2 and 3 are vacuum state, then the output state $\Psi_{out} = T\ \Psi_{in} = \sqrt{\frac{1}{3}}[|100\rangle + |010\rangle + |001\rangle]_{1'2'3'}$, where, 1', 2', 3' refer to three output ports of tritter. This is the W state which we want to produce. The probability of success is 100%. Obviously, besides one single-photon source and one common tritter, no other element is needed, so this scheme is very simple.

We can generalize this method to produce an arbitrary path W state of one photon. What we need is just a N×N lossless multiport fiber beam splitter. This multiport fiber coupler can be described by a unitary N×N matrix, where the matrix elements are the probability amplitudes for transmission from a certain input port to one of the N output ports. The relation between N input ports $[a_1, a_2, ...a_N]$ and N output ports $[b_1, b_2, ...b_N]$ can be described by the equation [16]

$$\begin{pmatrix} b_1 \\ b_2 \\ . \\ . \\ . \\ b_N \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & ... & ... & M_{1N} \\ M_{21} & M_{22} & ... & ... & M_{2N} \\ & & . & & \\ & & . & & \\ & & . & & \\ M_{N1} & M_{N2} & ... & ... & M_{NN} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_N \end{pmatrix}. \quad (2)$$

If the input state is $\Psi_{in} = [1, 0, ...0]_{1,2,....N}$, i. e., besides input 1 is in one photon state, the other N-1 inputs are in vacuum states , then the output state $\Psi_{out} = [M_{11}, M_{21}, ...M_{N1}]$, i. e., $\Psi_{out} = M_{11} |10...0\rangle + M_{21} |01...0\rangle + ... + M_{N1} |00...1\rangle$, where, $|M_{11}|^2 + |M_{21}|^2 + ... + |M_{N1}|^2 = 1$. So, by choosing a simple multiport fiber coupler, the kind of arbitrary path W state with one photon can be produced very easily. The successful probability is 100%.

Next, we consider how to generate the multiphoton polarization W state. For application, this kind of W state is more useful. We also take how to produce the three-photon W state $\frac{1}{\sqrt{3}}[|HHV\rangle + |HVH\rangle + |VHH\rangle]$ as an example, the generalization to the arbitrary multiphoton W state $\alpha |HH.....HV\rangle_N + \beta |HH.....VH\rangle_N + ...+ \nu |VH.....HH\rangle_N$ is straightforward, where, H means horizontal linear polarization, V means vertical polarization, $|\alpha|^2 + |\beta|^2 + ... + |\nu|^2 = 1$. In order to generate the three-photon W state, we need three single photon sources, which produce H_1, H_2 and V_3 photons respectively and a 3×3 fiber tritter. Here, we use the symmetry fiber tritter. The transformation matrix is shown in Eq. (1). We let three photons 1, 2 and 3 enter the inputs 1, 2 and 3 of fiber tritter respectively at the same time. In outputs of tritter, if we only consider the case in which there is only one photon in each output, then we can get

$$[e^{i\frac{2\pi}{3}} + e^{i\frac{4\pi}{3}}][|HHV\rangle + |HVH\rangle + |VHH\rangle]_{1'2'3'}, \quad (3)$$

where, 1', 2' and 3' refer to three outputs of tritter. The probability of getting the W state is $\frac{1}{9}$, which is larger than the scheme in Ref. [15]. For producing four-photon W state $\frac{1}{2}[|HHHV\rangle + |HHVH\rangle + |HVHH\rangle + |VHHH\rangle]$, we can use a 4×4 canonical quarter to do it. [16]. By postselection, we can get it with probability 1/16, which is just few less than 2/27 of Ref.[15], but it is more simpler and more easy to do it. In principle, we can generate an arbitrary multiphoton W state by same way. What we need is just a N×N fiber coupler which can be described by the unitary transformation matrix Eq. (2) and N single photon sources. In practice, the N×N fiber coupler is standard commercial product, so the problem is single-photon source. Now, there are some methods to produce the single photon.[17-19]

In summary, we give two very simple schemes to produce two kinds of W states. These scheme just need a common commercial multiport fiber coupler and single photon sources, they are very easy to realize in practice. We can use this scheme to get the path W state with 100% probability without further requirements like post-selection. Combining with postselection, we can get multiphoton polarization W state probabilisticly. Of course, we can produce the more modes W state from less modes W state or more photons W state from less photons W state by using the same procedure as entanglement swapping [20], the disadvantage is that the Bell state measurement is needed, which makes this scheme difficult in practice.

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