Explicit Implementation of Quantum Circuits on Unidirectional Periodic Structure

Yasuhito Kawano[†], Shigeru Yamashita[†,††] and Masahiro Kitagawa[*,†,**]
[†]NTT Communication Science Laboratories, NTT Corporation
3-1, Morinosato Wakamiya Atsugi-shi, Kanagawa, 243-0198 Japan
[*]Graduate School of Engineering Science, Osaka University
[††]Quantum Computation and Information, ERATO (JST)
[**]CREST, Japan Science and Technology Corporation

1 Introduction

It is widely believed that quantum computers can be much more powerful than today's computers. However, it is also believed that the normal quantum computation model, where we can perform any unitary operation onto any pair of qubits, cannot be realized by the current technologies. Another quantum computation model, which is considered to be more realizable and scalable, has been proposed [1]. The model assumes arrays of weakly coupled quantum systems where an interaction exists only between neighboring qubits, and we can only perform the same quantum operation onto all the qubits (we will mention formally in the next section.). Although this model seems to be feasible for realization, we cannot realize a normal quantum circuit directly, i.e., we need to translate a normal quantum circuit description into the specific description for the model. In this abstract, we summarize what logic operations can be performed in the model, and present an efficient strategy to translate a normal quantum algorithm into a sequence of the logic operations for the model.

2 Logic Operations on Unidirectional Periodic Structure

In [1], a heteropolymer, $ABCABC \cdots ABC$, in which each unit possesses an electron that has a long lived excited state, is considered for the realization of quantum computation. For each unit, A, B or C, call the ground state 0, and the excited state 1. Since the excited states are long lived, the transition frequencies ω_A, ω_B and ω_C between the ground and excited states are well-defined. In the absence of any interaction between the units, it is possible to drive transitions between the ground state of a given unit, say B, and the excited state by shining light at the resonant frequency ω_B on the polymer. It is supposed that there are local interactions between the units of the polymer. The effect of such interactions is to shift the energy levels of each unit as a function of the energy levels of its neighbors, so that the resonant frequency ω_B, for instance, takes on a value ω_{01}^B if the A on its left is in its ground state and the C on its right is in its first excited state. If the resonant frequencies for all transitions are different for different values of a unit's neighbors, then the transitions can be driven selectively: if a π pulse with frequency ω_{01}^B is applied to the polymer, then all the B's with an $A = 0$ on the left and a $C = 1$ on the right will switch from 0 to 1 and from 1 to 0. If all transition frequencies are different, these are the only units that will switch. Driving transitions selectively by the use of resonant π pulses induces a parallel logic on the states of the polymer: a particular resonant pulse updates the states of all units of a given type as a function of its previous state and the states of its neighbors. All units of the given type with the same values for their neighbors are updated in the same way. That is, applying a resonant pulse to the polymer effects the action of a cellular automaton rule on the states of units of the polymer.

Here we generalize the above ABC structure to the model that we call *unidirectional periodic structure*:

(1) The structure consists of repetitions of m qubits, each of which is expressed as $A_1 A_2 \cdots A_m$. Below, for the simple notation, we sometimes refer to A_1 in the next repetition as A_{m+1}.

(2) The interaction between qubits exists only between neighboring qubits.

(3) We can perform only the same operation to the same kind of qubits.

For the logical operations, we consider the following two operations:

(1) A operation which performs CNOT type operation whose target bit is A_i and the control bit is A_j (A_j must be the neighbor of A_i) in all the sequences, i.e., CNOT gates are performed onto all the pair of A_i and A_j at the same time.

(2) A operation which performs CCNOT type operation whose target bit is A_i and the control bits are A_j and A_k (A_j, A_i and A_k must be placed in this order) in all the sequences.

By using the above operations, we can perform the followings:

(1) A swap operations between A_i and A_{i+1} in all the sequences. This can be done by three CNOT type operations.

(2) A controlled swap operation that swaps A_i and A_{i+1} depending on the state of the adjacent qubit (A_{i-1} or A_{i+2}). This can be done by two CNOT type operations and one CCNOT type operation.

3 Implementation of Quantum Circuits on Unidirectional Periodic Structure

It is well known that any quantum circuit can be constructed from one-input quantum gates and CNOT gates. Accordingly, our essential task is to transform a CNOT gates whose control and target bits are not adjacent into a sequence of the above logic operations. Although Lloyd showed that it is possible to move quantum states in the unidirectional periodic structure as we desire [2], it has not yet been clear that how we can perform a CNOT gates whose control and target bits are not adjacent on the unidirectional periodic structure. Below we propose a method which transforms a CNOT gate in a quantum circuit with n variables into $O(6n-3m)$ logic operations on the unidirectional periodic structure.

Our method initially assigns each input of a given quantum circuit, say $x_1, \cdots, x_n$, to each qubit in the repetition of $A_1 A_2 \cdots A_m$ one by one, but we do not use A_t ($1 \leq t \leq m$). One A_t is set to be 1, whereas the other A_t is set to be 0. We call the only one A_t (which is set to be 1) the *control bit*. In other words, we use $m-1$ qubits in one repetition for storing original data, and one A_t in all the repetitions for the control bit. Below, for simplicity, we suppose $n = k(m-1)$ where k is an integer and $m \leq n+1$ (generalization is easy). That is, we need k repetitions to store all the data.

Next, we explain how we construct a sequence of logic operations corresponding to an original CNOT gate whose control and target bits are x_i and x_j, respectively. We suppose x_i and x_j are now on A_l and A_p, respectively, and $l \neq p$. (If $l = p$, we can transform the CNOT gate into fewer logic operations.) We say x_i and x_j are in the same group if they are on the same type of A_k. We can shift all the data in the same group by one repetition of m qubits relatively to the other groups by swapping data $m-1$ times. (This can be done $3(m-1)$ operations.) We just say this operation "shift the group by one repetition."

Step 1. Suppose that the repetition which has x_i be r repetitions away from the repetition which has the control bit. Then shift the x_i group by r repetitions. r is at most $k-1$, therefore, we need at most $3(m-1)(k-1)$ operations for this step.

Step 2. Suppose that the repetition which has x_j be h repetitions away from the repetition which has the control bit. Then shift the x_i group by h repetitions. We also need at most $3(m-1)(k-1)$ operations for this step.

Step 3. Now, x_i, x_j and the control bits are within m qubit range. We need at most $(m-3)$ swaps ($3(m-3)$ operations) to move x_i and x_j to be placed adjacently to the control bit.

After Step 3., the three qubits, x_i, x_j and the control bit, are placed adjacently. Therefore, we can perform the original CNOT gates by using the controlled swap operation.

Our procedure transform a CNOT gate into a sequence of fewer number of logic operations than what was implicitly proposed in [2]. We think our method probably optimal in terms of transforming each CNOT gate into a sequence of the logic operations on the unidirectional periodic structure. However, the order of the original CNOT gates is considered to have a great impact on the transformation result by our method. Therefore, we should consider how to order the original CNOT gates before applying our method, which is our future work.

References

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