# Quantum Stag Hunt Game and 2× 2 Symmetric Games with Four parameteres in Payoff bimatrix

Norihito Toyota

Hokkaido Information University.
59-2 Nishinopporo Ebetsu City Hokkaido 069 JAPAN
E-mail:toyota@do-johodai.ac.jp

In this paper, I study quntize some famous 2× 2 symmetric games with four parameteres in payoff bimatrix, especially the stag hunt game, according to the quantization scheme for game theory proposed by Marinatto et al.[2] where they mainly has discussed the Battle of Sexes game (BS game) with only 3 paremeters in the payoff bimatrix, and I make some discussion on them.

The stag hunt game is a non-zero sum game that the payoff bimatrix is represented as $2 \times 2$ symmetric bimatrix given by Fig.1, where the strategy $C$ means cooperation and the $D$ means defection[1]. Players are Bob and Alice as referred in the usal game theory and their payoffs are represented as $\$_A$ and $\$_B$, respectively. $a, b, c, d$ are real numbers with the conndition $a > b > c > d$. It is known that this game has a dilemma[3].

| | | Bob | |
|---|---|---|---|
| | | C | D |
| Alice | C | $(a, a)$ | $(d, b)$ |
| | D | $(b, d)$ | $(c, c)$ |

Fig. 1. Payoff function in the stag hunt game.

The rewards $\$ = \$_A = \S_B = \$$ at Nash Equilibria of the game are shown in Table1 1 with $m = \frac{c-d}{a-b+c-d}$, where $p$ and $q$ are the possibilities that Alice and Bob choose the strategy $C$. The following relations also hold good among estimated rewards;

$$\$(1,1) = a > b > \$(m,m) > c = \$(0,0) > d. \tag{1}$$

| $p = 1,\ q = 1$ | $p = 0,\ q = 0$ | $p = q = m$ |
|---|---|---|
| $a$ | $c$ | $\frac{ac-bd}{a-b+c-d}$ |

Table 1. Payoff functions of Nash Equilibria in the stag hunt game.

When an initial state (strategy) is a factorizable state, we can find three Nash Equilibria exactly same as ones in the classical case, which is given by Table 1. Interesting cases are when entangled states, which is essentialy the peculiar to quntum mechanics, are taken as an initial state;

$$|\psi_{in} >= \alpha|CC > +\beta|DD > \quad with\ |\alpha| + |\beta| = 1. \tag{2}$$

By calculating the payoff functions according to Marinatto et al.[2], we obtain the following equations;

$$\Delta_A \equiv \$_A(p^*, q^*) - \$_A(p, q^*) \quad = \quad (p^* - p)\left[q^*(a + c - b - d) + |\alpha|^2(d - c) + |\beta|^2(b - a)\right], \tag{3}$$

$$\Delta_B \equiv \$_B(p^*, q^*) - \$_B(p^*, q) \quad = \quad (q^* - q)\left[p^*(a + c - b - d) + |\alpha|^2(d - c) + |\beta|^2(b - a)\right]. \tag{4}$$

From these, we find three Nash Equilibria as shown in Table 2 where $m_q = \frac{(c-d)|\alpha|^2+(a-c)|\beta|^2}{a-b+c-d}$.

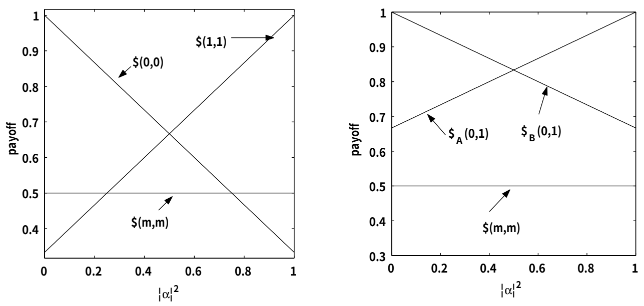


Figure 1: Payoff of the stag hunt game vs. $|\alpha|^2$ (left) and payoff of the Leader game vs. $|\alpha|^2$ (right).

| $p = 1,\ q = 1$ | $p = 0,\ q = 0$ | $p = q = m_q$ |
|---|---|---|
| $a\|\alpha\|^2 + c\|\beta\|^2$ | $c\|\alpha\|^2 + a\|\beta\|^2$ | $\frac{(ac-bd)+\|\alpha\|^2\|\beta\|^2(a+b-c-d)(a-b-c+d)}{a-b+c-d}$ |

Table 2. Payoff functions at Nash Equilibria in the stag hunt game for entangled case.

I explore what realtions among $\$(0,0)$, $\$(1,1)$, $\$(m_q, m_q)$ in the all parameter regions of $\alpha,\ \beta,\ a, b, c, d$ hold good generally by calculating $\$(1,1) - \$(m_q, m_q)$, $\$(0,0) - \$(m_q, m_q)$ and the standard results of the theory of quadratic equation. The results are schematically summarized in the left picture of Figure 1 which is rather complicate than the ones of the BS game with three parameters. From this, the degeneracy is not resolved in the same sense as the BS game. However, when the condition that the players always choose the maximum payoff solution among Nash Equilibria is imposed, the solution will be determined uniquely. Especially at $|\alpha|^2 = |\beta|^2 = \frac{1}{2}$, the same situation as the BS game occurs in the stag hunt game. Generally, however, the strategy with the best payoff depends on the initial quantum state, that is, $|\alpha|$, $|\beta|$. The classical case corresponds to the case with $|\alpha|^2 = 1$ where $p = q = 1$ give the best payoff for both players. On the other hand at $|\alpha|^2 < \frac{1}{2}$, the best payoff comes when $p = q = 0$ and such situation never occurs classically.

Since we give the general forms (3),(4) of payoff functions in $2 \times 2$ symmetric bimatrix game with the four parameters, we can a little discuss more general dilemma games with this form, (i)chicken game, (ii)deadlock game and (iii)prisoner's dilemma, referred in [3]. The prisoner's dilemma, however, shows so complicate aspect and the deadlock game is so simple that I discuss only (i)chicken games in a wide sense, including so-called chickin game($b > a > d > c$), leader game($b > d > a > c$) and hero game($d > b > a > c$)[1].

These three games have three Nach Equilibria; (i)$p = 1,\ q = 0$, (ii)$p = 0,\ q = 1$, (iii)$p = q = m - \frac{d-c}{b+d-a-c}$. When typical values are chosen for $a, b, c, d$, their payoff functions v.s. $|\alpha|^2$ are schematically shown in the left side of the Figure 1 where the exchange $\$(0,0) \rightarrow \$_A(0,1)$ and $\$(1,1) \rightarrow \$_B(0,1)$ are made (chickin and hero games) and the right side (leader game) where $A$ and $B$ in each sloping line must be exchanged for $(p,q) = (1,0)$, respectively. The essential results are the same as those of the stag hunt game and the difference between the left and right figures depens explicitly on parameters in payoff bimatrix and is not crusial. At $|\alpha|^2 = |\beta|^2 = \frac{1}{2}$ the payoff of Nash Equilibrium point is equal to be $\frac{1}{2}$ same as the classical case in the all three games. When getting out of this point, the degeneracy of $p = 1,\ q = 0$ and $p = 0,\ q = 1$ is resolved.

# References

[1] A. Rapoport and M. Guyer, "A taxonomy of 2 x 2 games", General Systems, vol.11, pp.203-214

[2] L. Marinatto and T. Weber, "A Quantum approach to Static games of Complete Information", Phys. Lett. A 272, 291 (200)

[3] W. Poundstone, "Prisoner's Dilemma; Jhon von Neumann, Game Theory, and the Puzzle of the Bomb", (New York, Doubleday 1995).