Role of entanglement for encoding and decoding quantum information

Mio Murao
Department of Physics, Faculty of Science, University of Tokyo,
7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
Tel & Fax: +81-(0)3-5841-4608
murao@phys.s.u-tokyo.ac.jp

Vlatko Vedral
Blackett Laboratory, Imperial College,
Prince Consort Road, London SW7 2BZ, United Kingdom
Tel: +44 (0) 207 594 7724   Fax: +44 (0) 207 823 8376
v.vedral@ic.ac.uk

Abstract

We present protocols for encoding and decoding via generalized teleportation in order to characterize encoding and decoding of quantum information represented by an unknown state. We discuss the role of entanglement for encoding and decoding quantum information.

We analyze the encoding and decoding of quantum information represented by an unknown single qubit state $|\phi\rangle = \alpha |0\rangle + \beta |1\rangle$. Encoding is a mapping from a single-qubit state $|\phi\rangle$ to a multi-qubit state $|\psi\rangle = \alpha |\psi_0\rangle + \beta |\psi_1\rangle$ where $|\psi_0\rangle$ and $|\psi_1\rangle$ are orthonormal multi-qubit states. Decoding is $|\phi\rangle$ to $|\psi\rangle$. Adding an ancilla qubit state $|0...0\rangle$ encoding process can be described by a unitary operation U_e as $|\psi\rangle = U_e |\phi\rangle \otimes |0...0\rangle$. In this case, decoding is described by $U_d = U_e^\dagger$, the reverse process. However the unitary operations U_e and U_d are over-specific for encoding and decoding – they include information about the ancilla system. Therefore, we consider encoding and decoding via generalized teleportation, which requires no information regarding the ancilla system, and try to characterize encoding and decoding by investigating the entanglement required for encoding and decoding.

We present protocols for optimal cloning type encoding (telecloning) and decoding (reverse-telecloning) via generalized teleportation. Surprisingly, one of the entangled states needed for the decoding process is a *single* bound entangled state, which had previously been considered "useless" for quantum information processing. We have also shown that the unlockable bound entangled state can be used for remotely decoding a distributed 3-qubit error correction state, which may be useful for the secure transmission of quantum keys.

We analyze the asymmetry of the encoding and decoding process from the viewpoint of required entanglement. Further we investigate an encoding scheme which is more asymmetric for encoding and decoding than telecloning and reverse-telecloning. In the extreme case, encoding requires entanglement and global operations but decoding can be performed using only local operations and classical communications without any entanglement. This strong asymmetry may suggest the possibility of quantum one-way function.

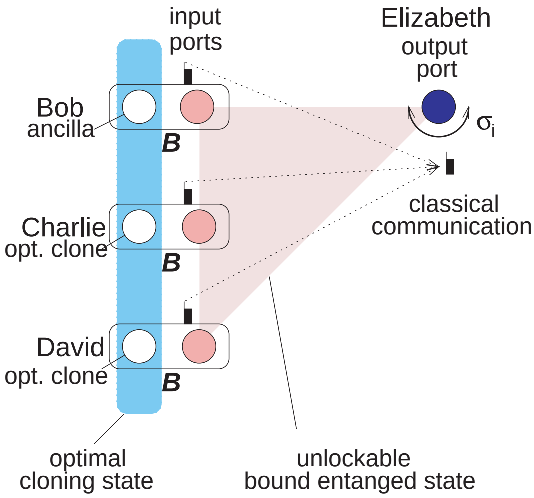

Figure 1: Schematic picture showing the decoding process (reverse-telecloning) from the three qubits shared by the spatially separated senders (Bob, Charlie and David) $|\psi_c\rangle_{BCD}$ to the remote receiver (Elizabeth)$|\phi\rangle_E$, using an unlockable bound entangled state ρ_{ub}.

Reference

M. Murao and V. Vedral, Phys. Rev. Lett. **86**, 352 (2001)