Fidelity of teleportation through noisy channels

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We investigate quantum teleportation through dissipative channels. Fidelity is calculated as a function of damping rates. It is found that the average fidelity of teleportation and the range of states to be teleported depend on the type and rate of the damping in the channel. We derive two bounds on the damping rates of the channels: one is to beat the classical limit and the second is to guarantee the security of the teleportation that is the non-existence of any other copy with better fidelity. We intend to show that priori information about the dissipative channel might be helpful for successful teleportation.

The quantum state of a system may be transmitted from a location to a distant one using only classical communication provided that the sender and the receiver share nonlocal entangled states. In a perfect scheme, the shared entangled state is maximally entangled enabling perfect quantum state transfer. However, in practice, entanglement is susceptible to noise due local interactions with the environment which may result in its loss of coherence. Noise can affect a teleportation process during the preparation and/or preservation of the state to be teleported, distribution and/or preservation of the entangled state, and during the measurement process. Generally, the source of the noise in a process is not exactly known. In some cases, one of the parties may try to measure the unknown state and/or clone it, or the channel through which the entangled state is distributed may introduce noise, or an eavesdropper may attack the channel, or a third party who prepares and distribute the entangled pair may fail to prepare a maximally entangled state. In this article, we will consider the teleportation of qubits of the form $|\varphi_{in}\rangle = \cos(\delta/2)\mathrm{e}^{i\gamma/2}|0\rangle + \sin(\delta/2)\mathrm{e}^{-i\gamma/2}|1\rangle$ in the presence of damping channels.

According to the definition given in [1], in the process of quantum teleportation, one can construct an exact replica of the original unknown quantum state with the cost of destroying the original one. Therefore, we can say that to call a quantum state transfer operation as quantum teleportation, the process should not only beat the classical limits but also obey the no-cloning theorem.

The resemblence of two quantum states and the properties of quantum state transfer are quantified by the fidelity, which measures the overlap of the states to be teleported $|\psi_{in}\rangle$ and the output state with the density operator $\hat{\rho}_{out}$, as

$$F = \langle\psi_{in}|\hat{\rho}_{out}|\psi_{in}\rangle. \tag{1}$$

The value of fidelity depends on the input state and on the noise acting on the qubits. The relation between the fidelity of teleportation process and the degree of entanglement shared by the parties have been studied by many researchers. Quality of the shared entangled state is a good criterion to quantify the reliability of the quantum teleportation. Bennett et al showed that less entangled quantum channel reduces the fidelity of teleportation and the range of states that can be teleported [1]. Horodecki et. al [2] have shown that for a shared bipartite entangled state $\hat{\rho}_{ent}$ to be useful for quantum teleportation, its fully entangled fraction f_{ent} given by

$$f_{ent} = max[\ \langle\Phi|\hat{\rho}_{ent}|\Phi\rangle\ ] \tag{2}$$

must be greater than 1/2. In Eq. (2), maximum is taken over all maximally entangled states $|\Phi\rangle$. It has also been shown that, the maximum achievable teleportation fidelity F is related to f_{ent} by $F = (2f_{ent} + 1)/3$. States with $f_{ent} \leq 1/2$ cannot be used for teleportation unless they are enhanced through filtering and distillation to satisfy $f_{ent} > 1/2$. Choosing the boundary value of $f_{ent} = 1/2$ gives a teleportation fidelity of $F = 2/3$. These values determines the boundary between classical and quantum state transfer. That is if $f_{ent} \leq 1/2$ and hence $F \leq 2/3$, then the same operation can be done classically. It has also been shown in the study of cloning that a state-independent universal $1 \to 2$ cloning machine has an optimum fidelity of $F = 5/6$ [3,4]. This implies that if one enforces the fidelity to be larger than this value, cloning operation cannot be done. If applied to teleportation, it can be said that if the fidelity of teleportation is larger than 5/6, then teleportation is secure in the sense that no other copy with the same or better fidelity can be found. This condition forces the f_{ent} to be larger than 3/4. If the security of quantum state transfer is concerned and quantum teleportation is to be used in the applications of secure quantum communication, one needs a measure to quantify the process and check whether it obeys the definition of the teleportation process given above.

Following the arguments stated above that $f_{ent} > 1/2$ must be achieved to beat the classical limit, and $f_{ent} >$

3/4 to meet the secure teleportation criterion, we obtained the bounds for damping rates of the channels. Moreover, we compare the fractional entanglement of Horodecki [2] and the Wootter's concurrence [5] for each channel to quantify the shared entanglement between Alice and Bob after the qubits are undergone dissipation. A comparison of these two measures are done and the corresponding value of concurrence of the shared entanglement to beat the classical limit and to ensure secure teleportation are obtained.

In the following we assume that both channels have the same damping rates, p. In the case of an amplitude damping channel, If only one of the qubits is affected by dissipation then $p < 2(\sqrt{2}-1)$ and $p < 2\sqrt{3}-3$ must be satisfied, respectively, to beat the classical limit, and to meet the security criterion, independent of the initial maximally entangled state. On the other hand, for the two qubit affected case, the security criterion forces $p < 1/4$ when the initial maximally entangled state is $|\psi\rangle = (|01\rangle \mp |10\rangle)/\sqrt{2}$ and $p < 1-\sqrt{2}/2$ when it is $|\phi\rangle = (|00\rangle \mp |11\rangle)/\sqrt{2}$. It is observed that for $|\phi\rangle$, the two-qubit affected case can be made to have higher f_{ent} than the one-qubit affected case and f_{ent} can be increased so that teleportation fidelity can beat the classical limit.

In the cases of phase damping and depolarizing channels, f_{ent} becomes independent of the initial maximally entangled state. For a phase damping channel, $f_{ent} > 1/2$ is satisfied for $\forall p, p \neq 1$ in one-qubit and two-qubit affected cases. To satisfy the security, the conditions on the damping rates are much stricter and can be written as $p < 1/2$ for one qubit-affected case, and $p < 1-\sqrt{2}/2$ for two-qubit affected case. On the other hand, for the depolarizing channel for which $p = 1$ corresponds to complete depolarization, $f_{ent} > 1/2$ is satisfied for $p < 2/3$ for one qubit-affected case, and $p < 1-\sqrt{3}/3$ for two-qubit affected case. $f_{ent} > 3/4$ is satisfied for $p < 1/3$ for one qubit-affected case, and $p < 1-\sqrt{6}/3$ for two-qubit affected case.

In addition to that we have analyzed the effect of damping parameters on the fidelity and average fidelity of teleportation. The range of states that can be teleported with fidelity values satisfying the above mentioned criteria are investigated. It is observed that average fidelity of teleportation may decrease below 2/3 down to 1/2 depending on the strength of damping in the amplitude damping and depolarizing channels (see Fig. 1). However, in the case of phase damping channel the minimum value of teleportation is 2/3 that is the boundary between classical and quantum operations. For state dependent teleportation process we have found the boundaries for the state angles (δ, γ) in connection with the channel damping parameter. For a phase damping channel with $p = 1/2$, if only one of the qubits is affected by damping, all the states $|\varphi_{in}\rangle$ can be teleported with $F > 2/3$, and only the states having $0.7\pi < \delta \leq \pi$ or $0 \leq \delta < 0.3\pi$ can be teleported with $F > 5/6$. When $\delta = 0$ or $\delta = \pi$, a teleportation fidelity of unity can be achieved. In the presence of an amplitude damping channel, the range of states that can be teleported correctly depend on the shared entangled state (whether it is initially $|\psi\rangle$ or $|\phi\rangle$) and the measurement result of Alice. If we assume that the maximally entangled state $|\psi\rangle$ is distributed through an amplitude damping channel with $p = 1/2$ and Alice's measurement result is $|11\rangle\langle 11|$, the range of states that can be teleported with $F > 5/6$ and $F > 2/3$ can be found, respectively, as $0 \leq \delta < 0.43\pi$ and $0 \leq \delta < 0.57\pi$. Fidelity of teleportation is state independent in the presence of depolarizing channel with a value of $F = 1-p(2-p)/2$ when both qubits are affected, and $F = 1-p/2$ when only one qubit is affected. This study shows that apriori information on the state to be teleported and the type and strength of dissipation can be exploited to achieve high fidelity teleportation to beat the classical limit and/or to meet the security needs.

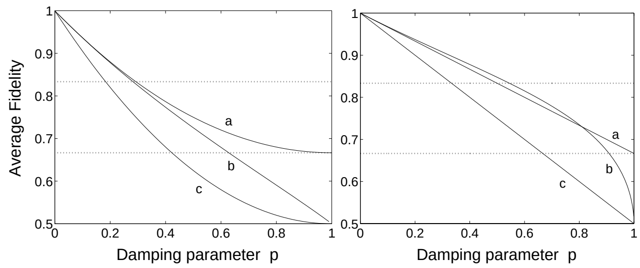

FIG. 1. Average fidelity of teleportation with noisy channels as a function of channel damping parameter: a, phase damping, b, amplitude damping, and c, depolarizing channel for the two-qubit affected scenario (left) and one-qubit affected scenario (right). Horizontal dotted lines denote the limits between classical and quantum operations (lower), and the secure quantum teleportation (upper).

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