Quantum Gates for Electrons Floating on Liquid Helium

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We investigate the construction of unitary quantum gates for the quantum-computing realization involving electrons floating on liquid helium. Platzman and Dykman established how to select the basis state for a qubit, how to manipulate the qubits, and how to measure the result of a computation. However, yet there exist no studies detailing exactly how the logical quantum gates would be constructed in this realization. Therefore, our goal is to show how one would actually perform the required unitary operations for a quantum computer using electrons floating on top of a thin superfluid helium film.

It is known that the set of all one-qubit gates and the CNOT gate are universal for quantum computation. Therefore, we discuss how to construct them. We also perform a numerical experiment to simulate both one-qubit gates as well as the CNOT gate. Since it becomes exponentially difficult to simulate larger qubit systems, only three qubits are studied, but additional qubits do not complicate the energy-level structure significantly if they are located far from the target qubit.

The electrons are confined to the surface of a helium substrate using a vertical electric field. Each electron has a separately controllable microdot electrode underneath it. The setting also requires the possibility of producing a uniform laser field which in general cannot be directed to particular qubits. The distance between the qubits is on the order of 1 μm. We assume the temperature to be low enough, $T \sim 10^{-2} K$; hence the system is coherent.

The qubit consists of the ground state and the first excited state of the electron in the z direction which we denote by $|0\rangle$ and $|1\rangle$, respectively. Logical quantum operations are performed by manipulating the matrix Hamiltonian of the qubit system. We include the Coulomb interaction between the electrons, the interaction between the qubits and a maser field and the energy difference between the states $|0\rangle$ and $|1\rangle$. Single-qubit and two-qubit gates can now be obtained by taking advantage of the Stark shift in the energy difference between the states $|0\rangle$ and $|1\rangle$ caused by an external microdot potential under each individual qubit and by the Coulomb interaction between the qubits. Qubits can then be addressed with a maser field in resonance with the transition frequency.

The read-out is accomplished by applying an inverse electric field $-E$ opposite to that pressing the electrons against the surface. The tunneling time away from the surface depends on the state of the electron. That is, if we measure the time that it takes for a qubit to decay, we get either a time that is characteristic of the ground state or of the first excited state. We are actually applying a set of projective measurement operators $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$. Decoherence in this system is mostly due to the coupling to the external microdot potentials and to thermally excited ripplons; Platzman and Dykman showed that the decoherence time is of order 10^{-4} s. The long decoherence time is a strength of this realization.

Applying a resonant laser field to a two-level electronic system results in Rabi oscillations. The frequency at which an uncoupled qubit would oscillate is $\hbar\Omega = |\langle 1|eE_\omega z|0\rangle|$, where we have assumed vertical polarization and E_ω is the microwave field amplitude. Applying a pulse of duration π/Ω would flip the qubit while a pulse of half this length would put the qubit in an equal superposition of $|0\rangle$ and $|1\rangle$. This operation, along with a phase shift, allows us to

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produce any unitary gate in $SU(2)$. In fact, any $U \in SU(2)$ may be represented in the form

$$U = \exp(i\gamma\sigma_z)\exp(i\beta\sigma_x)\exp(i\alpha\sigma_z). \tag{1}$$

It turns out that the z-rotation terms in Eq. (1) can be performed using the Stark shift and detuning the eigenfrequencies. The x rotation is actually performed using the Rabi oscillation.

When the microwave field is strong, we may consider the field as being just an external perturbation of the form $g\cos(\omega t)$. We write a simple Hamiltonian which governs the unitary time evolution of the n-qubit system by using a classical field:

$$\hat{H}(t) = \sum_{i=1}^{N}\left(-\frac{\hbar\omega_i}{2}\sigma_z^i + \sum_{\omega}\frac{g_\omega(t)}{2}\left[\cos(\omega t)\sigma_x^i - \sin(\omega t)\sigma_y^i\right]\right) + \frac{1}{2}\sum_{i\neq j} H_{\mathrm{int}}^{ij}. \tag{2}$$

The time-dependent part describes the maser field. The energy difference between $|0\rangle_i$ and $|1\rangle_i$ is $\hbar\omega_i$. The basis for the electrons is $|0\rangle = (1\ 0)^T$ and $|1\rangle = (0\ 1)^T$. The constant g is given by $g_\omega(t) = \langle 1|eE_\omega(t)z|0\rangle$ where $E_\omega(t)$ is the strength of the microwave field of the mode with the frequency ω. The interaction Hamiltonian H_{int}^{ij} for two qubits is given by the matrix elements $\langle kl|H_{\mathrm{int}}^{ij}|mn\rangle = -e^2\langle kl|(z_i - z_j)^2|mn\rangle/8\pi\epsilon_0 r_{ij}^3$, with $k,l,m,n \in \{0,1\}$.

If we want to do a rotation $\exp(i\alpha\sigma_z^i)$ on the i^{th} qubit, we simply detune the qubit using the microdot potential and keep the microwave field turned off. As we allow the time pass for Δt satisfying $\frac{\Delta t\delta}{2} = \alpha$, we get the desired gate. If the target qubit is the middle one (i=2), we may write the state vector as $|\psi(t)\rangle_1 \otimes |\psi(t)\rangle_2 \otimes |\psi(t)\rangle_3$. There are obviously four different frequencies corresponding to the transitions $|\psi(t)\rangle_1 \otimes |0\rangle_2 \otimes |\psi(t)\rangle_3 \to |\psi(t)\rangle_1 \otimes |1\rangle_2 \otimes |\psi(t)\rangle_3$ depending on the state of the first and third qubit because the system is asymmetric. This is because all the frequencies ω_i were distinct. In the double-chain model, there would be 6 principal qubits, i.e. 2^5 frequencies. Now if we select just these transition frequencies as our collection of frequencies, the theoretical unitary time-development operator corresponding to the situation is

$$U_x(\Delta t) = I \otimes \exp(\frac{i|g|\Delta t}{2}\sigma_x) \otimes I, \tag{3}$$

where the off-diagonal elements and the off-resonance effects were ignored. In any case, the frequencies can be read from the diagonal of the non-transformed Hamiltonian $\hat{H}_0$. We have chosen the microwave field strengths equal, such that $g = g_\omega(t)$ for all the four ω's in question. Choosing Δt so that $\frac{\Delta t|g|}{2\hbar} = \beta$ yields the desired result. In other words, using the four different frequencies means that we set explicitly

$$\begin{aligned} I \otimes \exp(i\beta\sigma_x) \otimes I = {} & |0\rangle\langle 0| \otimes \exp(i\beta\sigma_x) \otimes |0\rangle\langle 0| + |0\rangle\langle 0| \otimes \exp(i\beta\sigma_x) \otimes |1\rangle\langle 1| \\ & + |1\rangle\langle 1| \otimes \exp(i\beta\sigma_x) \otimes |0\rangle\langle 0| + |1\rangle\langle 1| \otimes \exp(i\beta\sigma_x) \otimes |1\rangle\langle 1| \end{aligned}$$

where each of the four terms in the sum is acquired with a different frequency microwave field.

Now we have explicitly shown how one-qubit operations can be realized for electrons on helium. As can readily be seen, the CNOT gate with for instance the first qubit as the control qubit and the second as the target, i.e., $|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes (|0\rangle\langle 1| + |1\rangle\langle 0|)$ can be performed by addressing two of the previously used four frequencies. Namely, we only use the microwave fields corresponding to the transitions $|1\rangle \otimes |0\rangle \otimes |0\rangle \to |1\rangle \otimes |1\rangle \otimes |0\rangle$ and $|1\rangle \otimes |0\rangle \otimes |1\rangle \to |1\rangle \otimes |1\rangle \otimes |1\rangle$ such that the CNOT is performed regardless of the state of the third qubit.

We could also do differently conditionalized operations with the four distinct transition frequencies. However, now that we can accomplish the CNOT and the operations in $U(2)$, we can in principle construct any unitary gate.

The simulation of the $SU(2)$ operations is a broader question since this group is infinite. We take as the function that we minimize $\|V - I \otimes U \otimes I\|_2$, where V is the actual physical time-development operator represented by an 8×8 matrix. We obtain the optimal parameters $\delta = 2.0331$, $\gamma = 1.0070$ which means that the chosen qubit separation of 0.5 μm is nearly optimal.

In summary, we show how to construct quantum gates for electrons floating on helium exploiting the Stark shift and a microwave field. We also show how to choose the parameters explicitly in order to get a desired gate. A quantum computer is universal if the CNOT and arbitrary operations in $U(2)$ are available: we show these operations to be available in this physical realization.

This realization of quantum computing is promising due to the cleanliness of the physical system; the system is also scalable. The long decoherence time allows for 10^5 single-qubit operations. Several experimental groups are working on this realization – provided that many-electron effects can be controlled, this realization may well produce a working quantum computer in the near future.