Manipulating quantum information of two trapped ions by a single-step operation

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Based on the exact conditional quantum dynamics for a two-ion system, here we propose an efficient *single-step* scheme for coherently manipulating quantum information of two trapped cold ions beyond the Lamb-Dicke limit by using a pair of synchronous laser pulses.

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I. INTRODUCTION

The entanglement between different qubits has recently become a focus of activity in quantum physics[1], because of experiments on non-local features of quantum mechanics and the development of quantum information physics. In the past few years, several key features of the proposal in [2], including the production of entangled states and the implementation of quantum controlled operations between a pair of trapped ions, have already been experimentally demonstrated[4–6]. Meanwhile, several alternative theoretical schemes[7–10] have also been developed for overcoming various difficulties in realizing a practical ion-trap quantum information processor.

II. MODEL

In this work, we propose an effective scheme for realizing the communication and logic operations between different trapped ions by *a single-step* operation, performed by using different laser beams synchronously. Following Sørensen and Mølmer [10], the Hamiltonian describing two trapped cold ions driven by a pair of synchronous laser beams is

$$\hat{H}(t) = \hbar\nu\left(\hat{a}^\dagger\hat{a}+\frac{1}{2}\right)+\hbar\omega_0\sum_{j=1}^{2}\frac{\hat{\sigma}_{z,j}}{2}+\frac{\hbar}{2}\sum_{j=1}^{2}\Omega_j\left\{\hat{\sigma}_{+,j}\ \exp\{i[\eta_j(\hat{a}^\dagger+\hat{a})-\omega_j t-\phi_j]\}+H.c.\right\}. \tag{1}$$

Under the rotating wave approximation, this system is exactly solvable *even if the usual Lamb-Dicke approximation is not made.* For brevity, the expressions for the solutions are omitted here and will be presented elsewhere.

III. RESULTS

Based on the exact solutions of the rotating wave approximation to the Hamiltonian (1), here we focus on how to realize in *one step* both two-qubit controlled operations and two-qubit entangled states between ion 1 and 2 beyond the Lamb-Dicke limit. This is achieved by properly setting up the controllable experimental parameters, e.g., the Lamb-Dicke parameters η_j, the carrier Rabi frequencies Ω_j, the frequencies ω_j ($j = 1, 2$) and duration of the applied synchronous pulses.

1. The $\hat{C}^Z$ gate

$$\hat{C}^Z_{12} \;=\; |g_1\rangle\,|g_2\rangle\,\langle g_1\,|\langle g_2| \;+\; |g_1\rangle\,|e_2\rangle\,\langle g_1\,|\langle e_2| \;+\; |e_1\rangle\,|g_2\rangle\,\langle e_1\,|\langle g_2\,| \;-\; |e_1\rangle\,|e_2\rangle\,\langle e_1\,|\langle e_2|, \tag{2}$$

can be realized directly by using a pair of red-sideband pulses.

2. if a resonant pulse is applied to ion 2 and a off-resonant pulse is simultaneously applied to ion 1, we find that the two-qubit controlled operation

$$\hat{C}_{12} \;=\; |g_1\rangle\,|g_2\rangle\,\langle g_1\,|\langle g_2| + |g_1\rangle\,|e_2\rangle\,\langle g_1\,|\langle e_2| \pm i\,e^{-i\phi_2}\,|e_1\rangle\,|g_2\rangle\,\langle e_1\,|\langle e_2\,| \pm i\,e^{i\phi_2}\,|e_1\rangle\,|e_2\rangle\,\langle e_1\,|\langle g_2|, \tag{3}$$

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can be implemented directly. These gates are equivalent to the exact CNOT gate, except for a local rotation.

3. Two-qubit entangled states

$$|\psi_{12}^{-}\rangle = U\,|g_1\rangle|e_2\rangle - V\,|e_1\rangle|g_2\rangle, \qquad |\psi_{12}^{+}\rangle = V\,|g_1\rangle|e_2\rangle + U\,|e_1\rangle|g_2\rangle, \tag{4}$$

and

$$|\phi_{12}^{-}\rangle = U\,|g_1\rangle\,|g_2\rangle - V\,|e_1\rangle\,|e_2\rangle, \qquad |\phi_{12}^{+}\rangle = V\,|g_1\rangle\,|g_2\rangle + U\,|e_1\rangle\,|e_2\rangle, \tag{5}$$

can be generated from the dynamical evolutions of the non-entangled initial states $|g_1\rangle|e_2\rangle$, $|e_1\rangle|g_2\rangle$ and $|g_1\rangle|g_2\rangle$, $|e_1\rangle|e_2\rangle$, respectively. These entangled states become the relevant two-qubit maximally entangled states, i.e., EPR states:

$$|\Phi_{12}^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|g_1\rangle\,|g_2\rangle \pm |e_1\rangle\,|e_2\rangle\right), \quad |\Psi_{12}^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|g_1\rangle\,|e_2\rangle \pm |e_1\rangle\,|g_2\rangle\right), \tag{6}$$

if the experimental parameters are set up properly.

4. The EPR states, e.g., $|\Phi_{12}^{-}\rangle$ and $|\Psi_{12}^{+}\rangle$, can also be generated by sequentially using the a single-qubit rotation and the controlled operation introduced above,

$$\begin{cases} |m\rangle\,|g_1\rangle\,|g_2\rangle \xrightarrow{\hat{r}_1} |m\rangle \otimes \frac{1}{\sqrt{2}}\left(|g_1\rangle|g_2\rangle - i\,e^{-i\varphi_1}\,|e_1\rangle\,|g_2\rangle\right) \xrightarrow{\hat{C}_{12}} |m\rangle \otimes |\Phi_{12}^{-}\rangle, \\ |m\rangle\,|g_1\rangle\,|g_2\rangle \xrightarrow{\hat{r}_1} |m\rangle \otimes \frac{1}{\sqrt{2}}\left(|g_1\rangle|g_2\rangle, - i\,e^{-i\varphi_1}\,|e_1\rangle\,|g_2\rangle\right) \xrightarrow{\hat{C}'_{12}} |m\rangle \otimes |\Psi_{12}^{+}\rangle. \end{cases} \tag{7}$$

IV. CONCLUSIONS

Based on the exact conditional quantum dynamics for two trapped ions driven by a pair of synchronous laser beams, we have shown that, under certain conditions, the quantum controlled gates and entanglement between a pair of trapped ions can be realized deterministically by *only a single-step operation.* The CM mode of ions always remains in its initial quantum state after the operation.

Compared to other approaches for coherently manipulating a pair of ions, the present scheme has some advantages. First, compared to the the previous multi-pulse schemes for two-qubit controlled gates (see, e.g.,[2, 8, 9]), the present single-step operational scheme may be easily tested experimentally, as the undesired dynamical phase evolution[11] related to the delay time between different operations is avoided completely. It may be more important that the present scheme can be used to describe a laser-ion coupling of arbitary strength, while previous schemes (see, e.g., [2, 10]) based on the LD approximation are only effective in the weak-coupling regime.

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