Quantum computation by measurements

Debbie W. Leung[1] *

[1] *Institute of Quantum Information, MSC 107-81, Caltech, Pasadena, CA 91125, USA*

Abstract. We first review a general method to perform universal quantum computation by measurements only, based on teleportation. Then, we prove the universality of two particularly simple sets of measurements. The first universal set consists of five simple measurements that act on one or two qubits. The second universal set consists of one measurement that acts on four qubits.

Keywords: Quantum computation, universality, measurement

1 Introduction

The standard model of quantum computation [1] requires a well defined and isolated Hilbert space. Universal computation further requires the ability (1) to prepare a fiducial initial state, (2) to implement a *universal* set of gates, and (3) to perform strong measurements. A set of quantum gates is universal if any unitary evolution can be approximated with arbitrary precision by a circuit involving those gates only [2]. For example, the following sets of gates are universal: (1) all 2-qubit gates, (2) all 1-qubit gates and the controlled-NOT (CNOT), (3) the CNOT, the phase gate, the Hadamard gate, and the $\frac{\pi}{8}$ gate, (4) almost any single 2-qubit gate.

Other computation models have been built upon the standard model, so as to achieve fault tolerance or to adapt to promising physical systems. In these models, only some unitary gates can be easily performed and they do not form a universal set. Methods have been developed to *indirectly* effect other gates to complete the universality requirement. These methods are all connected to teleportation [3, 4].

Recently, two quantum computation models that are based on measurements only have been proposed. Raussendorf and Briegel [5] have proposed a 1-way quantum computer that uses only 1-qubit measurements on a fixed initial state, which can be prepared by 4-qubit measurements. Independently, Nielsen [6] extended the indirect methods of performing unitary gates to achieve all requirements of the standard model using only measurements on up to 4 qubits. We call this "the teleportation model."

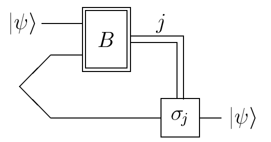
In this talk, we present various results in the teleportation model of quantum computation. We first review the general model that uses 4-qubit measurements only [6]. Then, we present our main results – the universality of two particularly simple sets of measurements. The first universal set consists of five simple measurements that act on one or two qubits. This shows that 2-qubit measurements are sufficient [7]. This is the minimal number of qubits to be measured jointly in order to achieve universality because measurements are the only means of interaction. The second universal set consists of one measurement that acts on four qubits. This corresponds to single 2-qubit gates that are proved universal in [8].

*wcleung@caltech.edu

2 Definitions and review of the teleportation model

Let $\sigma_{0,1,2,3}$ or I, X, Y, Z be the 2×2 identity matrix and Pauli operators. The Pauli group is generated by $\sigma_{0,1,2,3}$ acting on each qubit. The Clifford group consists of unitary operations that conjugate Pauli operators to Pauli operators. The Clifford group is generated by the CNOT, the phase gate P $= e^{-i\frac{\pi}{4}Z}$, and the Hadamard gate H $= \frac{1}{\sqrt{2}}(X+Z)$.

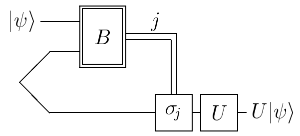
The crucial element in performing unitary gates indirectly is teleportation [3]:



(1)

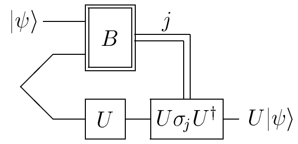
In the circuits throughout this abstract, time goes from left to right, single lines denote quantum registers, double lines denote classical data, single- and double-lined boxes respectively denote unitary gates and measurements. A gate connected to a measurement box by a double-line is conditioned on the measurement outcome. Two qubits connected at the left denote a maximally entangled state $|\Phi_0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. In the above circuit, B denotes the Bell measurement along the Bell basis $\{(\sigma_j \otimes I)|\Phi_0\rangle\}$.

To apply a gate U to a state $|\psi\rangle$, one can first teleport $|\psi\rangle$ and then apply U [4]:



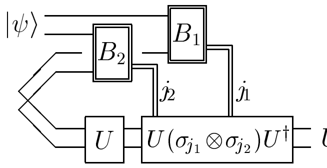
(2)

which implies the validity of the following circuit:



(3)

where $\dagger$ denotes the adjoint of an operator. Equation (3) is a recipe for an indirect implementation of U by preparing a state $(I \otimes U)|\Phi_0\rangle$ and applying a Bell measurement and a "correction" $U\sigma_j U^\dagger$. Equation (3) can be generalized to any n-qubit gate. For a 2-qubit gate U:



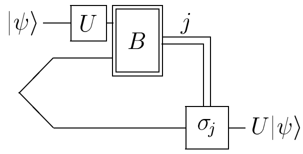
(4)

The nontrivial element in Nielsen's model is to perform a universal set of gates, such as all 2-qubit gates, by measurements only. A 1-qubit gate U can be performed using Eq. (3) – it suffices to prepare the state $(I \otimes U)|\Phi_0\rangle$ (by a 2-qubit measurement along the basis $\{(\sigma_j \otimes U)|\Phi_0\rangle\}$), perform the 2-qubit Bell measurement, and apply a correction $U\sigma_j U^\dagger$ when $j \neq 0$. With probability $1/4$, $j = 0$ and no correction is needed. If $j \neq 0$, the correction $U\sigma_j U^\dagger$ can be performed *recursively* using Eq. (3). This requires another correction gate with probability 3/4, again applied recursively. But in each application of Eq. (3), there is 1/4 chance that no more gate is needed, and the process is completed. On average this occurs after 4 trials. Any 2-qubit gate U can be performed analogously using Eq. (4). The 4-qubit ancilla can be prepared by a 4-qubit measurement.

3 Results

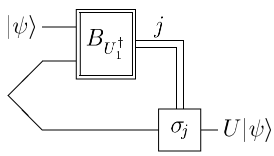
Our first main result is the universality of 2-qubit measurements [7]. In Nielsen's method, 4-qubit measurements are only needed in the ancilla preparation for 2-qubit gates. Our main observation is that only one 2-qubit gate is needed for universality. Choosing this to be the CNOT, the correction gates CNOT $(\sigma_{j_1} \otimes \sigma_{j_2})$ CNOT are just tensor products of Pauli operators (since CNOT is in the Clifford group). No other two-qubit gate needs to be implemented. Thus, 2-qubit measurements are sufficient, if one also has access to the ancilla for CNOT. In the talk, we will prove the universality of 2-qubit projective measurements by giving an explicit protocol to prepare the ancilla by measuring the (Hermitian) operators X, Z, $X \otimes X$ and $Z \otimes Z$.

Next, we present simple discrete universal sets of measurements using a new recipe in additional to Eqs. (3) and (4). One can apply a gate U and then teleport $U|\psi\rangle$:



(5)

The combined effect of applying U followed by the Bell measurement is equivalent to applying a measurement $B_{U_1^\dagger}$ along the basis $\{(U^\dagger \otimes I)|\Phi_j\rangle\}$. This gives a variant of Eq. (3):



(6)

The advantage of Eq. (6) is that the correction gate is always a Pauli operator (instead of $U\sigma_j U^\dagger$ as in Eq. (3)). It is known that {CNOT, H, P, U} is universal for any 1-qubit gate U outside the Clifford group. We will use Eq. (4) to implement the CNOT and Eq. (6) to implement the 1-qubit gates. Since all correction gates are in the Pauli group, we can enumerate all the required measurements:

$$Z, XX, ZZ, XZ, XY, (\mathrm{U}^\dagger X \mathrm{U}) \otimes X, (\mathrm{U}^\dagger Z \mathrm{U}) \otimes Z \,.$$

Choosing U$= e^{-i\frac{\pi}{8}Z}$ further simplifies the set to:

$$Z, XX, ZZ, XZ, \frac{1}{\sqrt{2}}(X+Y) \otimes X \,.$$

Our final result is to apply a generalization of Eq. (6) for any 2-qubit gate U to show that there exists single 4-qubit measurement that is universal. The main idea is that, a single measurement along the basis $\{(U^\dagger \otimes \sigma_{j_1} \otimes \sigma_{j_2})\, |\Phi_0\rangle_{13} \otimes |\Phi_0\rangle_{24}\}$ can be used in two different ways. First, it has the combined effect of applying U and two parallel Bell measurements. Second, it can prepare the ancilla $(U^\dagger \otimes I)|\Phi_0\rangle \otimes |\Phi_0\rangle$. In the talk, we will compose these two resources to efficiently perform a 2-qubit gate that is proved universal in [8].

References

[1] D. P. DiVincenzo, Quantum computation. Science **270**, 255 (1995).

[2] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information.* Cambridge, 2000.

[3] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. Wootters, Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels. Phys. Rev. Lett. **70**, 1895 (1993).

[4] D. Gottesman and I. Chuang, Quantum teleportation is a universal computational primitive. Nature **402**, 390 (1999).

[5] R. Raussendorf and H. J. Briegel, A One-Way Quantum Computer. Phys. Rev. Lett. **86**, 5188 (2001).

[6] M. A. Nielsen, Universal quantum computation using only projective measurement, quantum memory, and preparation of the 0 state. quant-ph/0108020v1.

[7] D. Leung, Two-qubit projective measurements are universal for quantum computation. quant-ph/0111122.

[8] A. Barenco *et al.*, A Universal Two–Bit Gate for Quantum Computation. quant-ph/9505016, 1995.