Implementing quantum noiseless coding using linear optics

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Abstract. We report an experimental implementation of Schumacher's noiseless quantum coding using single photons and linear optical elements. We use a single photon in a superposition of 4 paths and 2 linear polarization modes to represent a 3 qubit word comprising 2 path qubits and 1 polarization qubit. We use quantum coding to compress the 3 qubit word into 2 qubits and to decompress it back into 3 qubits. We present experimental fidelities for the coding-decoding operations.

Keywords: Quantum source coding, Linear optics

1 Introduction

Source coding concerns *how to represent messages as efficiently as possible*, being one of a fundamental issue in information theory. The unequal frequencies of the letters imply a redundancy that enables the compression of the message. When all alphabet appear with equal probability, then any compression is classically impossible. In quantum domain, there is another kind of redundancy when the letters are conveyed by the non-orthogonal states, $|\psi_A\rangle$, $|\psi_B\rangle$, $|\psi_C\rangle$, $\cdots$ with corresponding probabilities P_A, P_B, P_C, $\cdots$. Significantly, compression is possible here even if $P_A = P_B = P_C = \cdots$, in contrast to the classical case. The *quantum noiseless coding theorem* [1, 2] implies that by coding the quantum message in blocks of K letters, $KS(\hat{\rho})$ qubits are sufficient to encode each block in the limit $K \to \infty$, where $S(\hat{\rho})$ is the von Neumann entropy of the density operator $\hat{\rho} = \sum P_n |\psi_n\rangle\langle\psi_n|$ and $S(\hat{\rho}) \le 1$.

The compression of non-orthogonal data sets has significant practical advantages. For example, in long-haul optical communication channels one must deal with sequences of attenuated weak coherent pulses, that is, non-orthogonal states. Expensive quantum channel resources can be saved by compressing the sequences before storing or relaying to another channel. Here we report an experimental demonstration of the reliable communication of 3-qubit codewords over a 2-qubit quantum channel [3].

2 Quantum coding protocols

Our demonstration is based on the example given by Jozsa and Schumacher [2]. Imagine Alice needs to send Bob a message composed of binary letters, "+" and "−", represented by the letter states $|\psi_+\rangle$ and $|\psi_-\rangle$,

$$|\psi_\pm\rangle = \alpha|0\rangle + \beta_\pm|1\rangle \ . \tag{1}$$

Here $|0\rangle$, $|1\rangle$ are an orthonormal (computational) basis, $\beta_\pm = \pm\beta$, $\alpha^2 + \beta^2 = 1$, and for clarity we assume α and β are real numbers. Let the letter states occur with equal likelihood. When one practically handles blocks in finite length, compression-decompression is associated with finite errors. We then ask how faithfully one can compress sequences of non-orthogonal letter states.

Following [2] we use blocks of 3 letter states:

$$\begin{aligned} |B_{\mathbf{L}}\rangle &= |\psi_{L_1}\rangle \otimes |\psi_{L_2}\rangle \otimes |\psi_{L_3}\rangle \\ &= \alpha^3|000\rangle + \alpha^2(\beta_{L_1}|100\rangle + \beta_{L_2}|010\rangle + \beta_{L_3}|001\rangle) \\ &\quad +\alpha(\beta_{L_1}\beta_{L_2}|110\rangle + \beta_{L_2}\beta_{L_3}|011\rangle + \beta_{L_1}\beta_{L_3}|101\rangle) \\ &\quad +\beta_{L_1}\beta_{L_2}\beta_{L_3}|111\rangle \end{aligned} \tag{2}$$

where $\mathbf{L} = (L_1, L_2, L_3)$ and L_1, L_2 and $L_3 \in \{+,-\}$. The index $\mathbf{L}$ selects one of 8 possible letter state configurations. Alice first applies the unitary transformation $\hat{U}$ which leaves all computational bases states unchanged except for the mapping $\hat{U}|100\rangle = |011\rangle$ and $\hat{U}|011\rangle = |100\rangle$. Alice then makes a projection measurement of the first (leftmost) qubit in the computational basis. When the measurement results in the state $|0\rangle$, the 3 letter block is projected onto the most-likely state space Λ spanned by the states $|000\rangle$, $|011\rangle$, $|010\rangle$, and $|001\rangle$. Essentially, the coding protocols amount to a perfect transmission of the most likely parts of blocks, which can be conveyed by the remaining last two qubits. In the case of the measurement result $|1\rangle$, the 3 letter block is projected onto the unlikely space Λ' spanned by the bases $|100\rangle$, $|101\rangle$, $|110\rangle$, and $|111\rangle$. The likelihood of projecting onto Λ' is proportional to β which can be made very small.

3 Optical scheme

Fig. 2 shows an ideal single-photon linear optics implementation of the block coding scheme of the previous section using polarizing beam splitters (PBSs), $\lambda/2$ wave plates (HWPs) and photodetectors (PDs). One photon is used to represent the 3 qubits in terms of two location (L) qubits (the first 2 qubits) and a polarization (P) qubit (the third qubit). The paths labeled A, B, C and D represent the states $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$ of the first two qubits, respectively, and the polarization directions in the plane and perpendicular to the plane of the optical circuit represent the states $|0\rangle$ and $|1\rangle$ of the last qubit,

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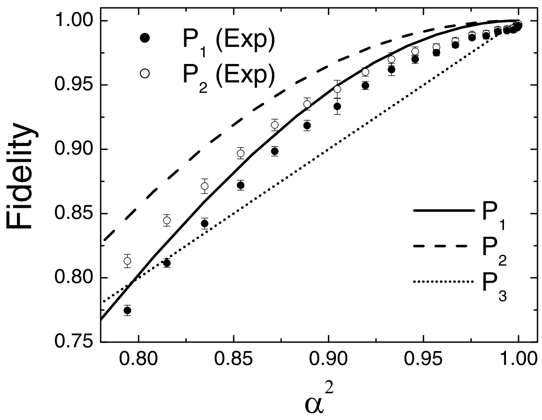

Figure 1: Comparison of the fidelities of various protocols as a function of the parameter α^2.

respectively. We parameterize the nth letter state by the angle $\theta_n = \frac{1}{2}\arcsin(\beta_{L_n})$.

A horizontally-polarized photon enters the quantum circuit at position E. The fully state-prepared block appears as a photon in a superposition of 4 path and two linear polarization modes along the vertical line labeled 3. This is the 3-qubit message Alice wants to compress and communicate to Bob.

The unitary transformation $\hat{U}$ is performed by the PBS and the two HWPs in the section between lines 3 and 4. The projection measurement of the first qubit is performed by the PDs D_1 and D_2 where the detection or non detection of a photon projects the state onto $|1\rangle$ or $|0\rangle$, respectively.

Now we consider two different protocols corresponding to two different actions taken when the block state is projected into the unlikely subspace Λ'. The first, P_1, is to treat the projection as an error and transmit no message and the second, P_2, is to transmit the coded word $|00\rangle$ by injecting an extra vertically-polarized photon into the top path. The fidelity for P_1 is $F_1 = \alpha^8(1+2\beta^2)^2$, and is plotted as the solid curve in Fig. 1. P_2 yields a higher fidelity than that of P_1, i.e.

$$F_2 = \alpha^8(1+2\beta^2)^2 + \alpha^6\beta^4(1+2\alpha^2) \ . \tag{3}$$

Finally, Jozsa and Schumacher also considered the simple protocol, P_3, where Alice discards the state of every first letter and sends the remaining 2 qubit letters to Bob, and Bob generates the state $|0\rangle$ for the missing letter state. This protocol yields an average fidelity of $F_3 = \alpha^2$, which is plotted as the dotted curve in Fig. 1.

The circuit in Fig. 2 implements protocol P_1 explicitly. Protocol P_2 can be implemented by switching a horizontally-polarized single photon source into the optical path A to encode the state $|000\rangle$ each time one of the photodetectors D_1 or D_2 detects a photon. The compressed 2-qubit message appears along vertical line 4. This is transmitted to Bob.

The decoding of the quantum channel AB at Bob's site requires a mirror image of the quantum circuit in Fig. 2

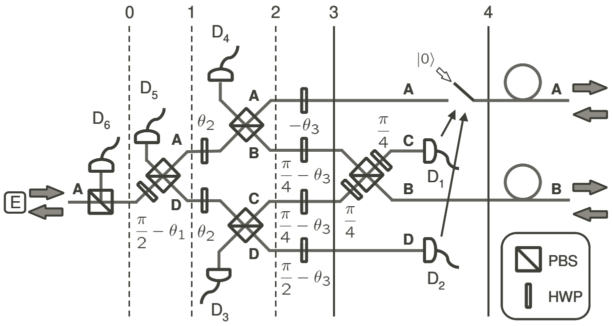

Figure 2: Linear optics implementation of three-qubit block coding and decoding.

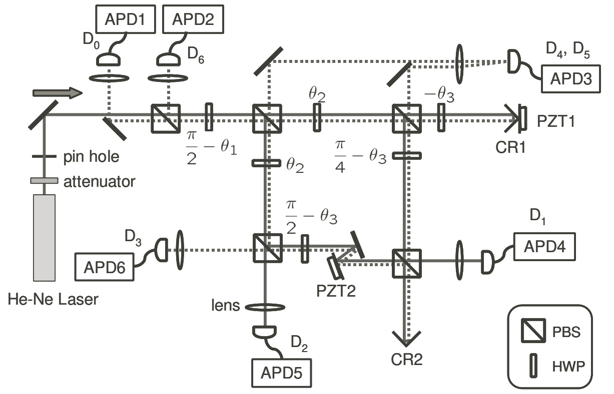

Figure 3: Experimental setup.

between lines 3 and 4 but without the PDs D_1 and D_2. Moreover, the mirror image of the circuit to the left of the line 3 can be used to determine the fidelity of the decoded block message. When the horizontally-polarized photon emerges from the mirror image of point E, then we can know that our compression-decompression succeeds.

4 Experimental Implementation

Our actual experimental circuit is shown in Fig. 3. Instead of constructing an additional mirror-image circuit for the decoding and fidelity check, we use corner reflectors (CR1 and CR2) to reflect the light in the quantum channel back through the circuit (shown as dotted lines in bthe figure). We use strongly attenuated light from a He-Ne laser (wavelength 632.8 nm) as our single photon source.

By varying the angle θ the fidelity is tested over a range of α values. The experimental fidelities for protocol P_1 and P_2 are shown in Fig. 1. The experimental fidelities for both protocols exceed that of the simple protocol P_3 (dotted line).

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