Nonlinear effects in bosonic channels

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Abstract. We analyze the role of nonlinear Hamiltonians in massless bosonic systems. We show that the information capacity as a function of the channel energy is increased, although only when the energy used for driving the nonlinearity is not considered as part of the energetic cost and when dispersive effects are negligible.

Keywords: Quantum communication, channel capacity, nonlinearity, quantum optics, entanglement, squeezing.

Noninteracting massless bosonic systems have been the object of extensive analysis [1, 2]. Their maximum capacity in transmitting information was derived for the noiseless case both in the narrow-band regime (where only few frequency modes are employed) and in the broad-band regime. However, it is still an open question whether nonlinearities in the system may increase these bounds: linearity appears to be the most important assumption in all previous derivations [2]. Up to now nonlinear effects have been used in fiber optics communications to overcome practical limitations, such as using solitons to beat dispersion or traveling wave amplifiers to beat loss [3]. The approach adopted in this paper is fundamentally different from these and from others where nonlinearities and squeezing are employed at the coding stage when using linear channels [4]. We follow the cue of a recent proposal [5] where interactions were exploited in increasing the capacity of a qubit chain communication line. In the case of linear bosonic systems, the information storage capacity of a signal divided by the time it takes for it to propagate through the medium gives the transmission capacity of the channel. In the presence of nonlinearities, dispersion can affect the propagation of the signal complicating the analysis, but an increase in the capacity can be shown, at least when the dispersive effects are negligible. A complete analysis of dispersion in nonlinear materials is impossible at this stage, since the quantization of these systems has been solved only perturbatively. The basic idea behind the enhancement we show is that the modification of the system spectrum due to nonlinear Hamiltonians may allow one to better employ the available energy in storing the information: we will present some examples that exhibit such effect. All these examples are highly idealized systems but are still indicative of the possible nonlinearity-induced enhancements in the communication rates. An important caveat is in order. In the physical implementations that we have analyzed, there is no capacity enhancement if we include in the energy balance also the energy required to create the nonlinear Hamiltonians. This is a general characteristic of any system: if one considers the possibility of employing all the available degrees of freedom to encode information, then one cannot do better than the bound obtained in the noninteracting case [6, 7]. However, the enhancement discussed here is not to be underestimated since there are situations in which some of the degrees of freedom are not accessible to encode information, but they can still be employed to augment the capacity of other degrees of freedom. A typical example is when the sender is not able to modulate the signals sufficiently fast to employ the full bandwidth supported by the channel: an external pumping (such as the one involved in the parametric down conversion case) may allow to increase the energy devoted to the transmission modes.

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References

[1] H. P. Yuen and M. Ozawa, Phys. Rev. Lett. **70**, 363 (1992).

[2] C. M. Caves and P. D. Drummond, Rev. Mod. Phys. **66**, 481 (1994), and references therein.

[3] H. A. Haus, *Electromagnetic Noise and Quantum Optical Measurements* (Springer, Berlin , 2000) and references therein.

[4] A. S. Holevo and R. F. Werner, Phys. Rev. A **63**, 032312 (2001); A. S. Holevo, M. Sohma, and O. Hirota, Phys. Rev. A **59**, 1820 (1999).

[5] S. Lloyd, Phys. Rev. Lett. **90**, 167902 (2003).

[6] V. Giovannetti, S. Lloyd, and L. Maccone, in preparation.

[7] M. Schiffer and J. D. Bekenstein, Phys. Rev. D **39** 1109 (1989).