Entanglement Distillation with Linear Optics

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Abstract. We present an entanglement distillation protocol for Gaussian states in the continuous variable regime. The iterative protocol can be carried out with weakly entangled pairs of light pulses as an initial supply, using beam splitters and photo-detectors. With ideal devices, the scheme can generate arbitrarily highly entangled states from a weakly entangled supply. We consider how the scheme will operate with imperfect devices and show that even here a large gain in entanglement can be achieved. Work on a practical implementation of this protocol is currently being pursued in collaboration with Ian Walmsley at the University of Oxford.

Keywords: Entanglement Distillation, Continuous Variables, Gaussian States, Linear Optics

Quantum information processing (QIP) in continuous variable (CV) systems is a promising counterpart to qubit-based QIP. A particularly useful class of CV states are the Gaussian states. Not only can these states be characterised theoretically in a particularly convenient way [1–3], they can also be generated and manipulated experimentally in a variety of physical systems, ranging from light fields [4] to atomic ensembles [5]. In a quantum information setting, entangled Gaussian states form the basis of proposals for teleportation [6] and cryptography [7].

Generating and maintaining entanglement between two distant parties, remains a unfulfilled goal of quantum information science. Entanglement purification protocols (also known as quantum privacy amplification) have been proposed for two-level systems (qubits) [8] but require, for their implementation, networks of quantum gates, which have so far only partly been implemented [9]. Given these difficulties, novel approaches are therefore welcome.

The most promising candidates for the transmission of quantum states are light pulses, which have both qubit (in their polarisation) and continuous variable (in amplitude and phase) behaviour. An alternative approach, then, would be an entanglement distillation protocol, based upon the continuous quantum variables of the quadratures of light pulses.

In this presentation, we will propose and analyse such a protocol [10] in a realistic experimental setting in preparation for a demonstration experiment.

1 Introduction

The talk will be structured as follows. First, as introduction, the concept of “entanglement distillation” and the basic quantum continuous variable properties of the light field will be discussed. Next, we will focus on Gaussian states and Gaussian operations. These states and the operations which map between them, are important both for their convenient theoretic description, and their practical importance, as most quantum states of light pulses produced in the laboratory are Gaussian, and Gaussian operations include many common components of linear optics networks. We then introduce an extremely important and surprising no-go theorem, namely, that Gaussian states cannot be distilled by Gaussian operations alone [1, 11, 12].

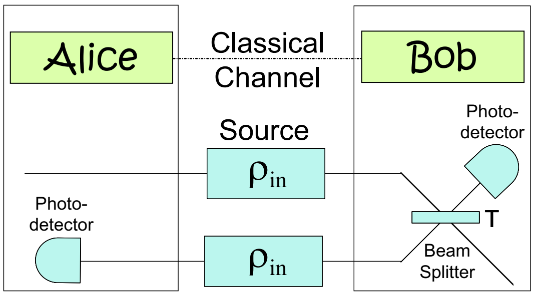

Figure 1: The initial non-Gaussian step of the protocol utilising two weakly entangled Gaussian states from the initial supply. Alice and Bob keep the remaining pair of pulses when both detectors indicate the presence of one or more photons. The choice of the beam splitter transmittivity leads to a particular non-Gaussian state being generated, depending on the initial supply.

In general non-Gaussian operations are extremely difficult to achieve. Consequently, the aim must be to devise schemes which employ non-Gaussian operations in a minimal way. In the following section we shall present an iterative entanglement distillation protocol, which, given an initial supply of weakly entangled Gaussian states, can produce a small number of states which are highly entangled and (approximately) Gaussian.

2 The Protocol

The protocol breaks down naturally into two components. The first part is a *single* step applied to pairs of Gaussian states from the initial supply and containing a non-Gaussian operation. We refer to this as the “non-Gaussian step” and it prepares a non-Gaussian state suitable for input into the next stage of the protocol. The non-Gaussian step is illustrated in Fig. 1

The second part of the protocol, illustrated in figure 2 is an iterative procedure which consists of Gaussian operations alone, which, after many iterations and under ideal conditions, generates, from a suitable input supply, a state which is very close to Gaussian. Analytic investigations of the convergence properties of this procedure have provided some interesting results [10, 15]. In particular, for any given input, mixed or pure, the limiting state generated after many iterations is

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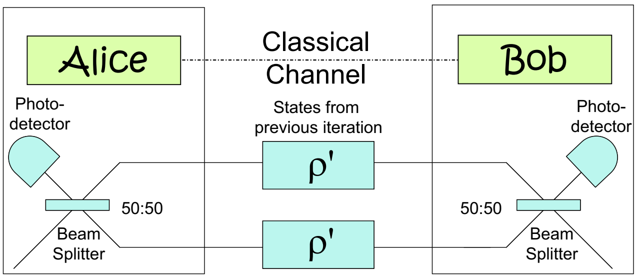

Figure 2: A single step of the iterative "Gaussification" stage of the protocol. Alice and Bob keep the remaining pair of pulses if their measurements both return a "no photon" outcome. Successful pairs are then used as inputs for the next iteration.

known. This state can be much more highly entangled than the input, and is always Gaussian, hence the name "Gaussification".

We will illustrate this for some typical states produced by the initial non-Gaussian step described above, and show that often, a large increase in entanglement can be achieved within a small number of iterations.

3 Realistic Devices

In a real implementation, operations can never be carried out perfectly and there will be many unavoidable experimental imperfections which could affect the scheme's operation. In the final parts of the talk we shall discuss the most important of these and the severity of their effects. In particular, at present, photo-detectors operating on the single-photon level will typically possess much less than unit detection efficiencies. Since the scheme relies on the distinction between the presence or absence of photons for its operation, one would imagine than inefficient detection could severely degrade the operation of the protocol. We shall present the results of numerical simulations of these effects and in light of these results and consideration of other potential imperfections the feasibility of an experiment to demonstrate the protocol will be discussed.

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