Quantum gate synthesis from flip-flop Hamiltonian

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Abstract. Based on the unitary transformation driven by a flip-flop interaction between a pair of qubits, a method is proposed to construct a controlled rotation around the x axis, from which quantum Fourier transformations are synthesized.

Keywords: NMR quantum computation, quantum gate, flip-flop Hamiltonian, QFT

1 Introduction

Liquid state NMR has been intensively used to demonstrate the quantum gates and algorithms experimentally [1, 2, 3]. One of the problems in NMR quantum computation in liquid is the difficulty of proper initialization due to extremely low nuclear polarizations. Because high nuclear polarization is achieved in solid state by dynamic nuclear polarization (DNP) using photo-excited triplet electron spin [4]. Initialization problem may be solved in solid state NMR quantum computation. However, the interactions between nuclear spins in solid are much different from those in liquid. Accordingly the strategy for constructing quantum gates could be different. In this paper, we propose a method for constructing a controlled rotation using flip-flop interaction in solid and its application to quantum Fourier transformations.

2 Flip-flop Hamiltonian

The interaction between a pair of nuclear spins I and S in solid under the rotational resonance experiment [5] is given by the flip-flop Hamiltonian.

$$\mathcal{H}_{\mathrm{ff}} = \hbar\chi(\sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+) \tag{1}$$

$$= \hbar\chi \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{2}$$

where $\sigma_\pm = \sigma_X \pm i\sigma_Y$. The corresponding time evolution operator $V(t)$ is given by

$$V(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\chi t & -i\sin\chi t & 0 \\ 0 & -i\sin\chi t & \cos\chi t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

Although $V(\frac{\pi}{2\chi})$ is a swap-like operation and $V(\frac{\pi}{4\chi})$ is sometimes called entangler, general effect of $V(t)$ is difficult to visualize. Therefore it would be very hard to construct quantum circuit based directly on $V(t)$.

3 Controlled rotation

Now let us consider a unitary transformation $W_x(\theta)$ defined by a quantum circuit shown in Fig. 1, where $R_x(\alpha)$ is a 1-qubit rotation by the angle α around the x axis.

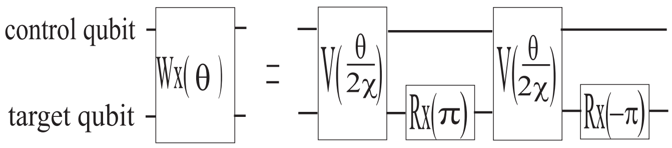

Fig. 1: A quantum circuit for $W_x(\theta)$.

W_x can be expressed in a matrix form as

$$W_x(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & 0 & 0 & -i\sin\frac{\theta}{2} \\ 0 & \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} & 0 \\ 0 & -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ -i\sin\frac{\theta}{2} & 0 & 0 & i\cos\frac{\theta}{2} \end{pmatrix}. \tag{4}$$

By examining the action of $W_x(\theta)$ upon the 2-qubit system, we have found that $W_x(\theta)$ corresponds to a unitary transformation rotating the target qubit by an angle $\pm\theta$ about the x axis when the control qubit points in the $\pm x$ directions (Fig. 2(a) and 2(b)). This operation is quite similar to the controlled rotation about z axis generated by scaler coupling in liquid state NMR.

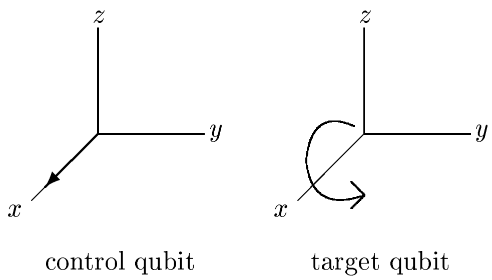

Fig. 2(a): When the control qubit is in the direction of x axis, the target qubit is rotated by θ around the x axis.

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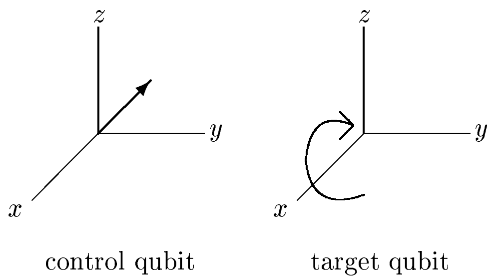

Fig. 2(b): When the control qubit is in the direction of $-x$ axis, the target qubit is rotated by $-\theta$ around the x axis.

4 Constructing QFT

Quantum Fourier transformation (QFT) [6] is the heart of Shor's quantum algorithm [7]. Two qubit gate required for QFT is a controlled phase rotation $B_z(\theta)$ about the z axis. Since controlled rotation about the x axis is easily obtained from flip-flop interaction, we can construct the x axis version of $B_z(\theta)$, defined by $B_x(\theta) = H^{\otimes 2} B_z(\theta) H^{\otimes 2}$ using $W_x(\theta)$ as shown in Fig. 3.

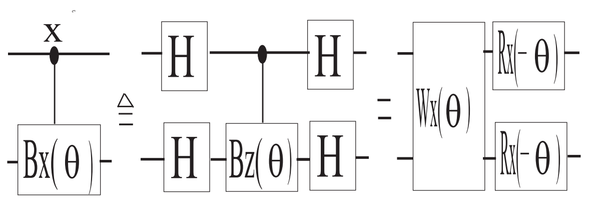

Fig. 3: A controlled rotation around the x axis. The target qubit is rotated by -2θ around the x axis only if the control qubit points in the $-x$ direction. "X" symbol at the control qubit indicates that the control is done in x basis rather than z basis.

Using $B_x(\theta)$, 2-qubit QFT is synthesized as shown in Fig. 4 and 3-qubit QFT is done in Fig. 5.

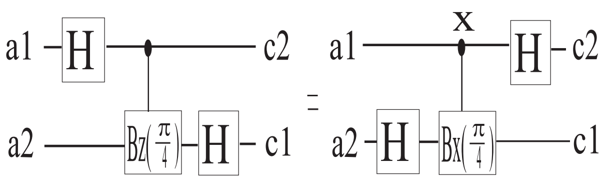

Fig. 4: A circuit for 2-qubit QFT

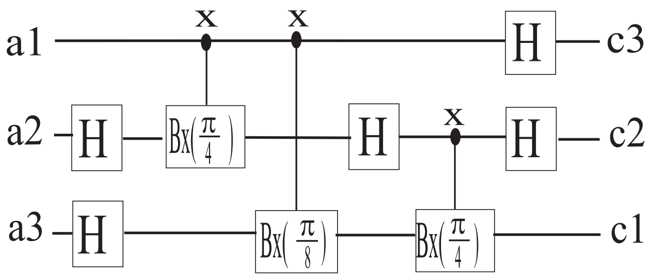

Fig. 5: A circuit for 3-qubit QFT

The number of elementary gates is essentially the same as the case of liquid state NMR.

5 Conclusions

Unitary evolution $V(t)$ generated by flip-flop Hamiltonian does not give an intuitive physical picture. However we can turn it into a controlled rotation about the x axis by applying a pair of π rotations about the x axis. The resulted gate $W_x(\theta)$ is equivalent to the controlled rotation about the z axis generated by J-coupling Hamiltonian common in liquid state NMR, except that the x axis and the z axis have been swapped. Therefore by moving into x basis and using $W_x(\theta)$ gates, quantum circuits such as CNOT [8, 9] and QFT can be constructed with the same simplicity and intuition. The scheme can be generalized for any axis in the x-y plane. The scheme might have applications to electron spin based quantum gates as well as solid state NMR.

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