The reduced dynamics in the presence of initial correlations

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Abstract. The reduced dynamics is considered in the presence of initial correlations. First, the inference given by Salgado and Sánchez-Gómez [1] is discussed in detail: The validity of the reduced dynamics with no initial correlations is examined in the presence of initial correlations. Our analysis shows that the reduced dynamics proposed by Štelmachovič and Bužek [2] must be adopted unless the joint dynamics is locally unitary. Secondly, we will characterize the difference of the dynamics between with and without initial correlations by taking a specific model, which implies the importance of initial correlations in the reduced dynamics.

Keywords: other topics

The reduced dynamics for open quantum system can be different from an unitary evolution: Generally, there exists an interaction between a system of interest S and its environment E, and this causes the reduced dynamics nonunitary. When studying the dynamics of an open quantum system, it is usually assumed that no initial correlations between system S and E exists: The density operator of the total system $S+E$ is *factorable* [3],

$$\rho_{\rm TOT} = \rho_{\rm S} \otimes \rho_{\rm E}, \tag{1}$$

where $\rho_{\rm S}$ and $\rho_{\rm E}$ are the density operators of the system S and E, respectively. This condition has been used in so many models for open quantum system [4], so that the reduced dynamics with no initial correlations has been well analyzed. In particular, the reduced dynamics with no initial correlations is always given by [4]

$$\rho_{\rm S}(t) = \sum_\mu W_\mu(t)\rho_{\rm S} W_\mu^\dagger(t) \quad \Big(\sum_\mu W_\mu^\dagger(t) W_\mu(t) = \mathbb{I}\Big), \tag{2}$$

the form of which is known as the Kraus representation [5, 6]. On the other hand, such a separation like Eq. (1) rather seems to be special case and more generally the reduced dynamics with initial correlations should be considered [7]. The central issue of this work is how such initial correlations give rise to change of the reduced dynamics (2).

In the first place, we review the results in Refs. [2, 1], generalizing them to include infinite dimensional cases and clalifying the meaning of them [8]. We introduce the correlation operator $\rho_{\rm COR}$ defined as

$$\rho_{\rm COR} \equiv \rho_{\rm TOT} - {\rm tr}_{\rm E}\rho_{\rm TOT} \otimes {\rm tr}_{\rm S}\rho_{\rm TOT}. \tag{3}$$

There exists no correlations if and only if $\rho_{\rm COR} = 0$, hence, this operator clearly embodies a quantity of a correlation between system S and E using a suitable norm. Note that by its definition, correlation operator has a property

$${\rm tr}_{\rm E}\rho_{\rm COR} = 0. \tag{4}$$

Let the total system be initially in a state $\rho_{\rm TOT}(0)$ with $\rho_S \equiv {\rm tr}_E\rho_{\rm TOT}(0)$, $\rho_E = {\rm tr}_S\rho_{\rm TOT}(0)$, and initial correlation operator $\rho_{\rm COR} \equiv \rho_{\rm TOT}(0) - \rho_{\rm S}\otimes\rho_{\rm E}$. Notice that ρ_S, reduced density operator, is nothing but the initial state of system S. Decomposing the total system Hamiltonian $H_{\rm TOT}$ into the system, environment and interaction parts:

$$H_{\rm TOT} = H_{\rm S} \otimes \mathbb{I}_{\rm E} + \mathbb{I}_{\rm S} \otimes H_{\rm E} + V, \tag{5}$$

the total system evolves unitarily: $\rho_{\rm TOT}(t) = U_{\rm TOT}(t)\rho_{\rm TOT}(0)U_{\rm TOT}^\dagger(t)$, where $U_{\rm TOT}(t) = \exp(-iH_{\rm TOT}t)$. Then, the reduced dynamics reads

$$\begin{aligned} \rho_S(t) &= {\rm tr}_{\rm E} U_{\rm TOT}(t)\rho_{\rm TOT}(0)U_{\rm TOT}^\dagger(t) \\ &= {\rm tr}_{\rm E} U_{\rm TOT}(t)\rho_{\rm S}\otimes\rho_{\rm E} U_{\rm TOT}^\dagger(t) + \delta\rho_{\rm S}(t) \end{aligned} \tag{6}$$

where

$$\delta\rho_{\rm S}(t) = {\rm tr}_{\rm E} U_{\rm TOT}(t)\rho_{\rm COR} U_{\rm TOT}^\dagger(t). \tag{7}$$

The first term of Eq. (6) is the same as that in the case with no initial correlation, which will be expressed by Kraus representation (2). It is the existence of the second term, or Eq. (7), that will serve as a deviation in the reduced dynamics between cases with and without initial correlations [2]. However, when the joint dynamics is locally unitary:

$$U_{\rm TOT}(t) = U_{\rm S}(t) \otimes U_{\rm E}(t), \tag{8}$$

with $U_{\rm S}(t) = \exp(-iH_{\rm S}t)$, and $U_{\rm E}(t) = \exp(-iH_{\rm E}t)$, the deviation (7) vanishes for any correlation $\rho_{\rm COR}$ [1]:

$$\begin{aligned} \delta\rho_{\rm S}(t) &= {\rm tr}_{\rm E}\{U_{\rm S}(t)\otimes U_{\rm E}(t)\rho_{\rm COR}U_{\rm S}^\dagger(t)\otimes U_{\rm E}^\dagger(t)\} \\ &= U_{\rm S}(t){\rm tr}_{\rm E}\{U_{\rm E}(t)\rho_{\rm COR}U_{\rm E}^\dagger(t)\}U_{\rm S}^\dagger(t) \\ &= U_{\rm S}(t){\rm tr}_{\rm E}\{\rho_{\rm COR}(0)\}U_{\rm S}^\dagger(t) = 0, \end{aligned} \tag{9}$$

where use has been made of the cyclic property of the trace and Eq. (4). This means that there are no deviations in the reduced dynamics between with and without initial correlation. In other words, we can use the usual formalism of the reduced dynamics of the case with no initial correlations even if there exists an initial correlation. However, the local-unitary evolution is quite a special case in the reduced dynamcis. Notice that the case of local-unitary evolution is equivalent to that with no interaction between system S and E, i.e., $V = 0$. It is natural to ask whether there are other evolution with $V \neq 0$ than the local-unitary case where the same property, i.e., $\delta\rho_{\rm S}(t) = 0$ for any $\rho_{\rm COR}$ and t, holds. In

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Ref. [8], we negatively gave the answer: We can prove that there are no other dynamcis which has such a property like in the case of local-unitary evolution. Namely, if there exists an interaction V between system S and E, the deviation (7) generally does not vanish and we have to consider the effect of the initial correlation [9]. In the following we give a sketch of the proof. Let us assume that $\delta\rho_{\rm S}(t) = 0$, $\forall t \in \mathbb{R}$ and $\forall \rho_{\rm COR}$. Differentiating this equation with respect to t, we obtain

$$\mathrm{tr}_{\rm E}[\mathrm{H}_{\rm TOT}, \rho_{\rm COR}] = 0, \tag{10}$$

at $t = 0$. Since $\mathrm{tr}_{\rm E}[\mathrm{H}_{\rm S} \otimes \mathbb{I}_{\rm E}, \rho_{\rm COR}] = [\mathrm{H}_{\rm S}, \mathrm{tr}_{\rm E}\rho_{\rm COR}] = 0$ by Eq. (4) and $\mathrm{tr}_{\rm E}[\mathbb{I}_{\rm S} \otimes \mathrm{H}_{\rm E}, \rho_{\rm COR}] = 0$ from the cyclic property of trace, we have

$$\mathrm{tr}_{\rm E}[\mathrm{V}, \rho_{\rm COR}] = 0. \tag{11}$$

From an arbitrariness of $\rho_{\rm COR}$ in this eqation, one can conclude [10] that V has to vanish, which means the dynamics is locally unitary. In otherwords, there are no other dynamics with $V \neq 0$ which has a property like in the case of local-unitary evolution. Thus, we can say that initial correlations play an important role in the reduced dynamics in general situation $V \neq 0$.

Next we would like to discuss the difference of the dynamics between with and without initial correlations. Especially by taking a specific model [11], we consider various quantities such as the purity (or the entropy) of the system, the quantity of the correlation and so on.

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References

[1] D. Salgado and J. L. Sánchez-Gómez. Comment on "Dynamics of open quantum systems initially entangled with environment: Beyond the Kraus representation" [PRA 64, 062106 (2001)]. quant-ph/0211164, 2002.

[2] P. Štelmachovič and V. Bužek. Dynamics of open quantum systems initially entangled with environment: Beyond the Kraus representation. Phys. Rev. A., 64:062106, 2001.; 67:029902(E), 2003.

[3] J. Bouda and V. Bužek. Purification and correlated measurements of bipartite mixed states. Phys. Rev. A., 65:034304, 2002.

[4] C. W. Gardiner and P. Zoller. *Quantum Noise*, 2nd ed. Springer-Verlag, 2000.

[5] K. Kraus. *States, Effects and Operations: Fundamental Notations of Quantum Theory.* Springer-Verlag, 1983.

[6] R. Alicki and K. Lendi. *Quantum Dynamical Semigroups and Applications.* Springer-Verlag, 1987.

[7] L. D́. Romero and J. P. Paz. Decoherence and initial correlations in quantum Brownian motion. Phys. Rev. A., 55:4070-4083, 1997.

[8] H. Hayashi, G. Kimura, and Y. Ota. Kraus representation in the presence of initial correlations. Phys. Rev. A., 67:062109, 2003.

[9] Notice that even in the case with $V \neq 0$, there are specific initial correlation $\rho_{\rm COR}$ dependent on V which vanishes the deviation (7) [8]. However, such a case is a special case and generally the deviation does not vanish. On the other hand, in the case of local-unitary evolution, the deviation vanishes for any $\rho_{\rm COR}$.

[10] The rigorous proof is given in Ref. [8] for the case of arbitrary composite systems with finite levels.

[11] Such an argument has been already reported in Ref. [2], but we investigate it further.