# Decoherence Control on Ion Trap System

Akira Tokuse[1] * Shuichi Tasaki[1] Paolo Facchi[2] Saverio Pascazio[2]

[1] *Department of Applied Physics, Waseda University. 3-4-1 Okubo, Shinjuku-ku, Tokyo 169-8555, JAPAN.*
[2] *Dipartimento di Fisica, Università di Bari and Istituto Nazionale di Fisica Nucleare. Sezione di Bari, I-70126 Bari, ITALY.*

**Abstract.** The control of thermal decoherence via dynamical decoupling and via the quantum Zeno effect (Zeno control) is investigated for a model of trapped ion, where the dynamics of two low lying hyperfine states undergoes decoherence due to the thermal interaction with an excited state. Dynamical decoupling is a procedure that consists in periodically driving the excited state, while the Zeno control consists in frequently measuring it. When the control frequency is high enough, decoherence is shown to be suppressed. Otherwise, both controls may accelerate decoherence.

**Keywords:** Quantum Computation, Decoherence Control, Dynamical Decoupling, Quantum Zeno Effect, Ion Trap

## 1 Introduction

The essential ingredient for the efficiency of quantum algorithms and cryptography is the principle of superposition of states. The loss of purity (i.e., decoherence) of states would deteriorate the performance. Thus, the information carried by a quantum system has to be protected from decoherence.

Among several schemes proposed so far, we focus on the quantum dynamical decoupling. It was proposed by Viola and Lloyd[1], where the system is periodically driven with period $T_c$ in an appropriate manner so that the target subsystem is decoupled from the environment. It was shown[1, 2] that a complete decoupling is achieved in the $T_c \to 0$ limit, or the limit of infinitely fast control. However, as it is not possible to achieve the $T_c \to 0$ limit, its performance for nonvanishing $T_c$ should be investigated.

The key ingredient of dynamical decoupling is the continuous disturbance of the system, which suppresses the system-environment interaction. The situation is similar to the so-called quantum Zeno effect. When the measurement process is described by a multidimensional projection operator, frequent measurements restrict the evolution within each subspace specified by the projection operator and a superselection rule dynamically arises[3]. Therefore, if one can design the measurement process so that different superselection sectors (defined by the given measurements) are coupled by the interaction between a target system and the environment, the system-environment interaction can be suppressed by frequent measurements. We refer to such a decoherence control as a *quantum Zeno control*.

In this presentation[4], the dynamical decoupling control and the quantum Zeno control are discussed for a model of the NOT gate which consists of a trapped ion.

## 2 Model and Decoherence

We consider a model of the trapped Be ion. The ion is assumed to be at rest and only the dynamics of the hyperfine states is considered. Then, the main mechanism of decoherence can be attributed to the emission and absorption of thermal photons associated with transitions to nearby excited states. For the sake of simplicity, one of the hyperfine states is assumed to couple electromagnetically with a nearby excited state (see Fig. 1), the polarization of the photon is neglected and only the rotating terms are taken into account (rotating wave approximation) for the driven parts.

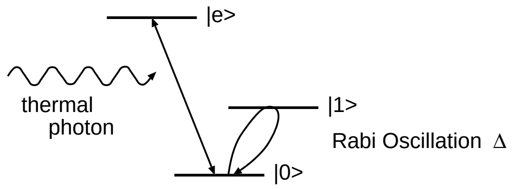


Figure 1: A schematic picture of the system. $|0\rangle$, $|1\rangle$ and $|e\rangle$ are the lower hyperfine, upper hyperfine and excited states, respectively.

We calculate the fidelity F of the target system with respect to the ideal NOT gate operation. It is plotted in Fig. 2 for the state starting from $\sigma(0) = |0\rangle\langle 0|$. As time goes on, the fidelity $F$ decreases, or decoherence takes place.

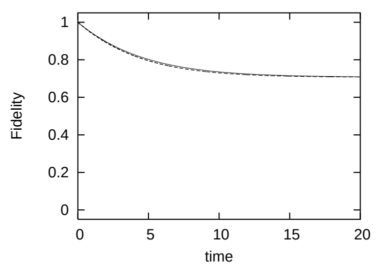


Figure 2: Time evolution of the fidelity $F$ of the target states. The time unit on the horizontal axis is the decoherence time.

*tokuse@ruri.waseda.jp

## 3 Control via Dynamical Decoupling

We consider a dynamical decoupling control of the thermal decoherence. Since decoherence arises from the transition between the states $|0\rangle$ and $|e\rangle$ it is expected to be suppressed if $|e\rangle$ does not contribute to the $|0\rangle$-$|1\rangle$ dynamics. So we consider a control via the Rabi oscillation between the state $|e\rangle$ and a higher excited state $|c\rangle$ (cf. Fig. 3).

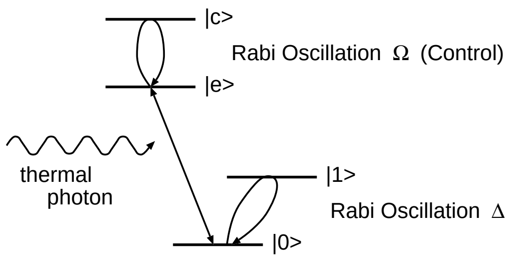


Figure 3: A schematic picture of the system under the quantum dynamical decoupling control.

The evolution of the fidelity of the target system is shown in Fig. 4 for different values of control parameters, and the other parameters are chosen as in the uncontrolled case except the ones related to the new level $|e\rangle$. Fig. 4 shows that the dynamical decoupling control may accelerate decoherence if the parameters are not appropriately tuned.

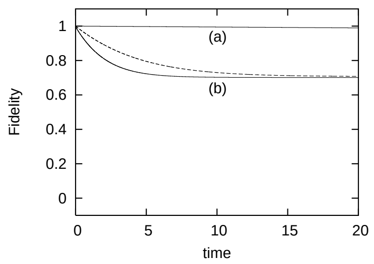


Figure 4: Evolution of the fidelity $F$ of the system state. The time unit in the horizontal axis is the decoherence time for the uncontrolled case. (a) Control frequency ($\sim\ \Omega$) is much higher than the frequency of $|0\rangle - |e\rangle$ transition:$\omega_e'$ (b) control frequency is of the same order of $\omega_e'$. For comparison, the behavior of $F$ without control is also displayed (broken curve).

## 4 Quantum Zeno Control

For the same reason as in the dynamical decoupling, we disturb the evolution of $|e\rangle$ by repeated measurements , once per $T_c$ (Fig. 5). The nonselective measurement of $|e\rangle$ causes the following change of the density matrix:

$$\rho \to \hat{P}\rho \equiv \pi_e \rho \pi_e + (\mathbf{1} - \pi_e)\rho(\mathbf{1} - \pi_e) \qquad (1)$$

where $\pi_e$ is a projection operator acting on the whole Hilbert space $\pi_e \equiv |e\rangle\langle e| \otimes \mathbf{1}_B$ and $\mathbf{1} = \mathbf{1}_S \otimes \mathbf{1}_B$.

The evolution of the fidelity is shown in Fig. 6 for different values of $2\pi/T_c$, where the parameters are chosen

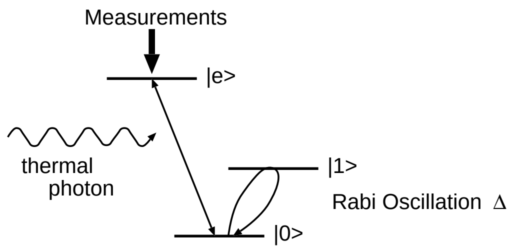


Figure 5: A schematic picture of the system under the quantum Zeno control.

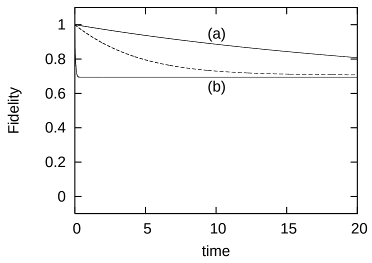


Figure 6: Evolution of the fidelity $F$ of the target states. The time unit on the horizontal axis is the decoherence time for the uncontrolled case. (a) Control frequency $2\pi/T_c = 5 \times 10^6 \times \omega_e'$; (b) control frequency $2\pi/T_c = 0.5 \times \omega_e'$. For comparison, the behavior of $F$ without control is also displayed by a broken curve.

as in the uncontrolled case. Fig. 6 shows that the Zeno control may accelerate decoherence if the parameters are not appropriately chosen.

In summary, we have studied the dynamical decoupling and Zeno controls for a model of trapped ions. By very rapidly driving or very frequently measuring the excited state, decoherence is shown to be suppressed. However, if the frequency of the controls are not high enough, the controls may accelerate the decoherence process and may deteriorate the performance of the quantum state manipulation.

## References


[1] L. Viola and S. Lloyd. Dynamical suppression of decoherence in two-state quantum systems. *Phys. Rev. A* 58:2733-2744, 1998.

[2] L. Viola, E. Knill and S. Lloyd. Dynamical Decoupling of Open Quantum Systems. *Phys. Rev. Lett.* 82:2417-2421, 1999.

[3] P. Facchi and S. Pascazio. Quantum Zeno subspaces and dynamical superselection rules. quant-ph/0207030, 2002.

[4] S.Tasaki, A.Tokuse, P.Facchi and S.Pascazio. Control of Decoherence: Dynamical Decoupling versus Quantum Zeno Effect - a case study for trapped ions. *Intl. J. of Quantum Chem.*, 2003 in press; quant-ph/0210129, 2002.