Preparation of Multi-Atom Greenberger-Horne-Zeilinger and W States in Cavity QED

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Abstract. A method to prepare multi-particle entanglement of GHZ-type and W-type is proposed through the interaction between the cavity and many atoms simultaneously. After the suitably interaction, the selected measurements on the cavity field can create the entangled atomic states. These entangled states can be useful for quantum information manipulation and fundamental study of quantum theory. These states that violate the Bell's inequality are proven.

Keywords: multi-particle entanglement, GHZ-type, W-type, Bell's inequality, cavity QED

1 Introduction

In recent years, generation of entanglement states has been studied intensely due to its applications in quantum communication and information processing such as quantum teleportation [1], super-dense coding [2], quantum key distribution [3], and telecoloning [4]. It was shown [5] that much stronger refutations of local realism can be provided by entangled states involving three or more particles. Some methods to prepare the entangled states of two or more particles in cavity QED has been proposed . Cirac and Zoller [7] proposed a method of generating three-particle GHZ state. The state is prepared by injecting the atoms sequentially through a suitably prepared single-mode resonant cavity. The cavity must be initially in the vacuum. After passage of the atoms, the cavity is left again in the vacuum. Gerry [8, 9] has proposed a scheme for preparing two or more atoms in macroscopically separated entangled states. After suitably prepared atoms interact with the cavity field a subsequent measurement on this field projects the atoms onto the entangled states [8]. If it involves two micromaser cavities each supporting two modes initially prepared in a nonlocalized two-photon state by an atom undergoing a two-photon transition, four atoms in a form of $|\varphi\rangle = \frac{1}{\sqrt{2}}\left[|+\rangle_1|+\rangle_2|-\rangle_3|-\rangle_4 \pm |-\rangle_1|-\rangle_2|+\rangle_3|+\rangle_4\right]$ can be prepared [9]. Zheng and Guo[10] have proposed a scheme for the generation of two-atoms maximally entangled state with cavity QED. In this paper, an alternative method is proposed to generate the GHZ state and W state of four particles with

$$\begin{array}{l} |\varphi\rangle_{GHZ} = \frac{1}{\sqrt{2}}\left(|0000\rangle + |1111\rangle\right) \\ |\varphi\rangle_W = \frac{1}{2}\left(|1000\rangle + |0100\rangle + |0010\rangle + |0001\rangle\right) \end{array} \tag{1}$$

After the selected measurements on the cavity field, one can easily get various kinds of three or four particle entangled states (GHZ state and W state). And then we will prove four-partical entangled states violate Bell's inequality.

2 Preperation of four-particle W state and GHZ state

Recent advances in cavity QED [13-17], both in the microwave and in the optical regimes imply that these states can be realized with existing and planned technology. This fact, together with the near-unity detection efficiency for Rydberg atomic states, makes the technology a promising way to prepare the entangled states [17]. By laser excitation and microwave manipulation, the cavity can be initially prepared in a coherent state of large amplitude

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle \tag{2}$$

The four atoms are initially prepared in a superposition of ground state $|g\rangle_i$ and excited state $|e\rangle_i$.

Consider four two-level atoms simultaneously interacting with the cavity field. In the limit of large number photon [22,23], the interaction between the cavity and atoms is much stronger than the coupling between the atoms, we can neglect the crossed terms corresponding to the interaction between the atoms, so the effective interaction Hamiltonian can be written as

$$H' = \eta a^+ a\left(\sigma_z^1 + \sigma_z^2 + \sigma_z^3 + \sigma_z^4\right) \tag{3}$$

Where $\sigma_z^i = |e\rangle_{ii}\langle e| - |g\rangle_{ii}\langle g|$, $\eta = \lambda^2/2\Delta$ is a parameter, λ is the atom-cavity coupling strength, $|\Delta| = |\omega_a - \omega_c|$ is the detuning between atom and cavity, ω_a is the atomic transition frequency and ω_c is the frequency of the cavity mode, a^+ and a are the creation and annihilation operators for the cavity field. Assuming that the detuning is so large that only virtual transition occur between levels $|e\rangle_i$ and $|g\rangle_i$, the dispersive interaction leads to phase shifts of the initial coherent state.

Then by the selected measurement on the field, we can reduce the atom-field state to the atomic entangled states. Assuming that measurements are made in a time short enough so that the cavity dissipation effects can be ignored. So the detection of the state $|-\alpha\rangle$ produces the GHZ state of four atoms

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$$|\varphi\rangle_{GHZ} = \frac{1}{\sqrt{2}}\left(|g_1 g_2 g_3 g_4\rangle + e^{i(\theta_1+\theta_2+\theta_3+\theta_4)}\,|e_1 e_2 e_3 e_4\rangle\right) \tag{4}$$

The detection of the state $\left|\alpha e^{i\frac{\pi}{2}}\right\rangle$ produces the W state of four atoms.

For circular state of Rydberg atoms and microwave cavities, both radiative lifetimes and cavity decay time are ranged from 0.01 to 0.1s. The required interaction time between atom and the cavity mode is $\leq 10^{-5}$ s. Thus there is sufficient time to prepare the states without decoherence due to dissipation [18]. Since cavity dissipation effects are assumed to be small during the atom-field interaction times, we can neglect the dissipation during the preparation process. So this scheme to prepare the atomic entangled states is valid.

3 Violation of the Bell inequality

Though it can be seen that these quantum states should violate Bell's inequalities, the violation of the Bell' inequality need to be proved [18-21]. Using the method introduced in Ref. [5], one can prove that these entangled states which are generated by our scheme are inconsistent with the local hidden variable theories. For the four-partical W-state, we can calculate the value of the measurement

and select

$$\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4 \tag{5}$$

so the expectation value of measurement is

$$E_W^{\varphi} = \tfrac{1}{4}[-4\cos\theta_1\cos\theta_2\cos\theta_3\cos\theta_4+ \\ \sin\theta_1\sin\theta_2\cos\theta_3\cos\theta_4 + \sin\theta_1\cos\theta_2\sin\theta_3\cos\theta_4+ \\ \sin\theta_1\cos\theta_2\cos\theta_3\sin\theta_4 + \cos\theta_1\sin\theta_2\sin\theta_3\cos\theta_4+ \\ \cos\theta_1\sin\theta_2\cos\theta_3\sin\theta_4 + \cos\theta_1\cos\theta_2\sin\theta_3\sin\theta_4] \tag{6}$$

one can get

$$A(0)\,B(0)\,C(0)\,D(0) = -1 \tag{7}$$

$$\frac{A\left(\frac{\pi}{2}\right)}{A(0)} = \frac{B\left(\frac{\pi}{2}\right)}{B(0)} = \frac{C\left(\frac{\pi}{2}\right)}{C(0)} = \frac{D\left(\frac{\pi}{2}\right)}{D(0)} = 0 \tag{8}$$

It is manifest that the two calculated outcomes contradict with each other.

For the four-particle GHZ-state, the expectation value of the measurement outcome is

$$E_{GHZ}^{\varphi}(n_1, n_2, n_3, n_4) = \cos(\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4) \tag{9}$$

We select n_1, n_2, n_3 in x-y plane,

$$\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 = 0 \tag{10}$$

the calculated value of the consequence is

$$C(\Phi)\,D(\Phi) = C(0)\,D(0) \tag{11}$$

This equation is not mathematically contradictory, but physically the contradiction emerges since $C(\Phi)\,D(\Phi)$ cannot be constant.

It is clear that the W state and the GHZ state of four-particle exhibit properties that can not be explained by the local hidden theory.

4 Conclusion

An alternative scheme to prepare two kinds of inequivalent multi-particle entangled states is presented. The scheme involves a single-mode quantized electromagnetic cavity and three and four two-level Rydberg atoms. The atoms interact with the cavity simultaneously. Through proper measurement on the cavity, one can get the atomic entangled states. It is shown that these states violate Bell's inequality. The atomic entangled state can be used to test the nonlocality by the use of classical microwave fields (Ramsey zones) followed by selective ionization.

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