Entanglement Generation by Quantum Chaos

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Abstract. Calculating the entropy of a reduced density matrix, we can recognize the entanglement generation in quantum chaotic bi-particle systems. According to the de Broglie-Bohm's interpretation of the quantum mechanics, in this paper, we reveal that quantum potential causes the suppression of chaos and the effect can be reduced in the multi-dimensional systems. Further, we try to qualify and quantify the entanglement induced by quantum chaos from the point of the dynamics of configuration volume elements.

Keywords: entanglement, quantum chaos, de Broglie-Bohm's interpretation

1 Introduction

In the theory of quantum computer and quantum information, a property inherent in quantum mechanics, entanglement becomes a target to be researched. At the dawn of the quantum mechanics, the entanglement received attention from the point of a paradox comprehended in the quantum mechanics[1]. In the theory of quantum open systems, the formation of entanglement between the relevant system and the irrelevant environment has also been considered as the origin of quantum decoherence.

It is shown that in the interacting bi-particles the entanglement between them is produced by the quantum chaos [2]. In the previous analysis, the entropy of a reduced density matrix has been used as a measure of entanglement. In order to reveal the mechanism of the entanglement generation induced by quantum chaos, in this paper, we describe the evolution of wave functions as rigid trajectories in the configuration space according to the de Broglie-Bohm's interpretation of quantum mechanics [3].

2 Coupled Non-linear System

As the quantum system with redundant degrees of freedom which are not necessary to cause chaos [4][2], we consider coupled two kicked rotors

$$H(\hat{q}_1, \hat{p}_1, \hat{q}_2, \hat{p}_2; t) \sum_{i=1,2} H_{k_i}(\hat{q}_i, \hat{p}_i; t) + c_{pp}\hat{p}_1\hat{p}_2. \qquad (1)$$

Here

$$H_k(\hat{q}, \hat{p}; t) = \frac{1}{2}\hat{p}^2 + k\cos(\hat{q})\sum_{n=1}^{\infty}\delta(t - n\mathrm{T}). \qquad (2)$$

Fig. 1 shows the time evolution of the linear entropy,

$$S \equiv Tr_1[(\hat{\rho}_{red})^2] \equiv Tr_1[(Tr_2(\hat{\rho}))^2], \qquad (3)$$

which is calculated for the reduced density matrix obtained by coarse-graining the second degree of freedom.

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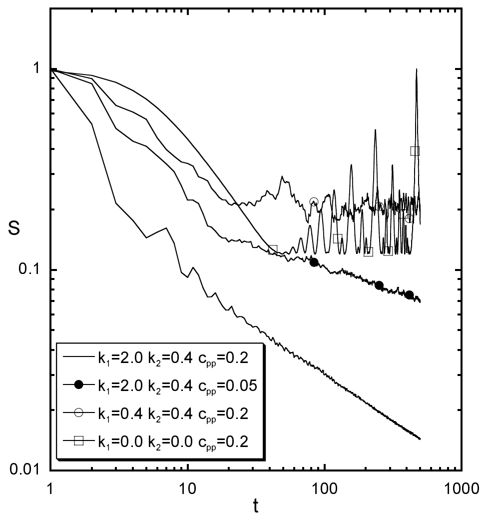

Figure 1: Variation of linear entropy S ($\hbar = 87\pi/4096 = 0.0667\cdots$)

According to the de Broglie-Bohm's picture, we depict the evolution of a quantum state as a bundle of rigid trajectories of the particles. First, we represent a wave function for the coupled kicked rotor, $\Phi(q_1, q_2; t)$ as a polar coordinate form,

$$\Phi(q_1, q_2; t) = R(q_1, q_2; t)\exp[\frac{\mathrm{i}}{\hbar}S(q_1, q_2; t)], \qquad (4)$$

where R and S are real-valued functions. Then, the Schrödinger equation for the system (1) is described as

$$\frac{\partial}{\partial t}R^2 + \frac{\partial}{\partial q_1}R^2((\frac{\partial S}{\partial q_1}) + c_{pp}(\frac{\partial S}{\partial q_2})) + \frac{\partial}{\partial q_2}R^2((\frac{\partial S}{\partial q_2}) + c_{pp}(\frac{\partial S}{\partial q_1})) = 0, \qquad (5)$$

$$\frac{\partial S}{\partial t} + \frac{1}{2}(\frac{\partial S}{\partial q_1})^2 + \frac{1}{2}(\frac{\partial S}{\partial q_2})^2 + c_{pp}(\frac{\partial S}{\partial q_1})(\frac{\partial S}{\partial q_2}) + V(q_1, q_2; t) + V_Q(q_1, q_2; t) = 0, \qquad (6)$$

where $V(q_1, q_2; t)$ is the original potential,

$$V(q_1, q_2; t) \equiv \sum_{i=1,2} k_i \cos(q_i) \times \sum_{n=1}^{\infty} \delta(t - n\mathrm{T}), \quad (7)$$

and $V_Q(q_1, q_2)$, what we called a quantum potential, is

$$V_Q(q_1, q_2; t) \equiv -\frac{1}{2R}(\sum_{i=1,2} \frac{\partial^2 R}{\partial^2 q_i} + 2c_{pp}\frac{\partial^2 R}{\partial q_1 \partial q_2})\hbar^2. \quad (8)$$

Next we introduce an ensemble of particles with the statistical distribution $R(q_1, q_2; t)^2$. If we assume that each particle moves with velocity

$$\dot{q}_1 = (\frac{\partial S}{\partial q_1}) + c_{pp}(\frac{\partial S}{\partial q_2}), \quad \dot{q}_2 = (\frac{\partial S}{\partial q_2}) + c_{pp}(\frac{\partial S}{\partial q_1}), \quad (9)$$

the distribution function $R(q_1, q_2, ; t)^2$ is conserved along the flow of the particles through Eq. 5. If we are given a wave function at arbitrary time, we can obtain a trajectory of a particle by integrating Eq. (9).

3 Entanglement and Mixing Property

According to the previous section, we describe the quantum dynamics in the bi-particle system (1) ($k_1 = 2.0, k_2 = 0.4$, $c_{pp} = 0.2$, and $T = 1.0$) as rigid trajectories in the configuration space.

First, we show the evolution of 20 particles from $t = 1.0$ to $t = 2.0$ (Fig. 2). The initial particles are settled along the two lines. The kick at $t = 1.0$ accelerates the particles in the region $q_1 \in [0, \pi]$ and decelerates ones which belong in the region $q_1 \in [\pi, 2\pi]$. This motion results in the high and low regions of the density of the particles. In the high density region, the quantum potential become prominent and scatter out the particles to flatten the density profile. The coupling in (1) induces the motion along to the q_2 axis. As a result, the lines along which the probe particles are connected are going to be bent. We note that from the point of the trajectory picture, the localization of wave functions in the single kicked rotor is reasonable. For the one-dimensional system, the quantum potential forbids the particles to go across each other and thus formation of the high-density region is strongly suppressed.

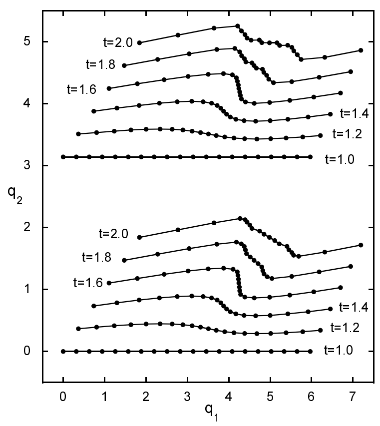

Figure 2: Quantum motion of probes initially set on a line ($q_2 = 0$ or π).

Next, we estimate the deformation of configuration volume element from $t = 30.0$ to $t = 31.0$. At $t = 30.0$, we settle uniformly 1,000,000 probes on the configuration space, $[0, 2\pi] \times [0, 2\pi]$ and plot the positions of the probes at $t = 31.0$ in Fig. 3, which indicates that the coupling utilizes the second dimension of the configuration space and results in the hard deformation of the volume elements. We note taht if the coupling is absent, the configuration volume, $\Delta q_1 \Delta q_2$ remains a rectangle.

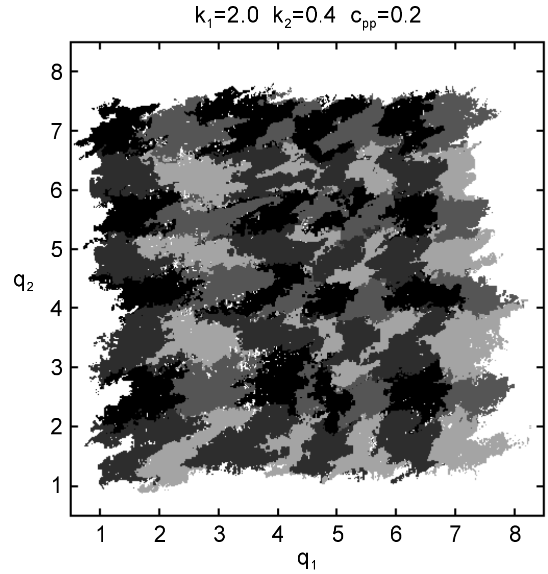

Figure 3: Deformation of configuration volume elements between the time interval $[30, 31]$.

4 Conclusion

We estimate the quantum dynamics that generate quantum entanglement in the bi-particle system from the point of the Bohm's trajectory picture. Comparing the parameter dependence of the entanglement generation (Fig. 1) with that of the deformation property of configuration volume elements, we can conclude that the entanglement generation in the bi-particle system corresponds to the mixing property of the two-dimensional configuration volume. The mixing in the multi-dimensional configuration space is induced by the stretched and folded motion by the quantum chaos.

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