Tailoring teleportation to the quantum alphabet

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Abstract. We introduce a refinement of the standard continuous variable teleportation measurement and displacement strategies. This refinement makes use of prior knowledge about the target state and the partial information carried by the classical channel when entanglement is non-maximal. This gives an improvement in the output quality of the protocol. The strategies we introduce could be used in current continuous variable teleportation experiments.

Keywords: continuous variable teleportation, quantum information theory, quantum optics, linear optics

Quantum teleportation has become a cornerstone of quantum information theory since its conception by Bennett *et al.* in 1993 [1]. It is a useful quantum information processing task both in itself, and as part of other tasks such as quantum gate implementation [2, 3]. In particular, optical implementations of teleportation [4, 5, 6, 7] may be useful in current linear optical quantum computing proposals [3].

Quantum teleportation is a process whereby the state of a quantum system can be communicated between two (possibly very distant) parties with prior shared entanglement, joint local quantum measurements, local unitary transformations and classical communication. In the standard scheme, the two parties are called Alice and Bob, and are sender and receiver respectively. Victor (the verifier) gives Alice a quantum system (the target) in a state known only to him. Alice makes joint quantum measurements on the target state and her part of the entanglement resource shared with Bob. The results of these measurements she shares with Bob via a classical communication channel. This information tells Bob the local unitary transformations he must perform on his part of the entanglement resource to faithfully reproduce the target at his location. Victor then compares the output state at Bob's location with the target state by calculating the overlap between the two. In its simplest form this is just the inner product of the two states and is in general known as the *fidelity*.

In ideal teleportation the resource is maximally entangled. As a result the classical channel carries no information about the target state. Also the alphabet of input states is assumed to be an unbiased distribution over the same dimensions as the entanglement. Examples of this include: the standard discrete protocol where qubits are both the target and entanglement resource [1]; and the original continuous variable protocol where the target is a flat, infinite dimensional distribution and the entanglement is idealised EPR states [8]. However, one may consider situations in which the entanglement is non-maximal and the alphabet of states is not evenly distributed. Additional information is now available prior to teleportation, from the restricted alphabet, and dynamically from the partial target information now carried by the classical channel. How should one then tailor the protocol so as to make best use of this additional information?

The situation arises naturally in practical implementations of continuous variable teleportation [9, 10, 5]. The entanglement resource most commonly used in continuous variable teleportation is the two-mode squeezed vacuum. It is not perfectly entangled, since this would require infinite energy. On the other hand an even distribution of target states is also unphysical. We are motivated to find ways in which to make maximum use of the resource given this situation. We outline here a general strategy and then describe a simple refinement of the standard continuous variable teleportation protocol which gives an improved output quality for a reduced alphabet of possible input states. It has the advantage that it may be implemented with currently available technology.

Consider the situation of teleporting coherent states. The state amplitudes will have an upper bound, and the probability of Victor preparing a state with a certain amplitude might be known. We consider three variations on this theme:

Two-dimensional Gaussian. The classical limit used in Ref. [5] and derived by Braunstein, Fuchs and Kimble [11] assumes that Victor produces coherent states with a symmetric two-dimensional Gaussian probability distribution, where coherent states of greater amplitude are less likely to occur than those with amplitude close to zero. The standard protocol assumes the width of this distribution is infinite. Braunstein, Fuchs and Kimble considered how the classical limit changed for finite width but not how to optimise the protocol as a function of this width. Choosing this smaller subset of states should allow Alice and Bob to improve the fidelity of their teleportation protocol.

Coherent states on a circle. Another possibility is that Victor could produce coherent states of an amplitude known to Alice and Bob, but of an unknown phase. If the amplitude of Victor's prepared coherent states is α, then these states will lie on a circle in phase space of radius α, hence the term "coherent states on a circle". This knowledge reduces the alphabet of possible output states substantially and

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should lead to a corresponding improvement in the fidelity.

Coherent states on a line. Conversely to coherent states on a circle, Victor could produce target states of known phase but unknown amplitude. These states would lie along a line in phase space and hence are termed "coherent states on a line". Again, the alphabet of states is reduced and the fidelity is expected to increase with respect to the standard protocol.

We now describe a general strategy for tailoring teleportation based upon maximising the fidelity over Bob's possible displacements in phase space. Another technique of fidelity optimisation has been discussed by Ide *et al.* [12], which uses gain tuning to improve the fidelity output. Our scheme is similar, however we use the one-shot fidelity of teleporting a coherent state to find Bob's optimum displacement. The technique described here gives very simple relations describing the displacement Bob must make to achieve the best possible fidelity given the level of squeezing, Alice's measurement results, and the known properties of the target state.

This technique can be improved further if one tailors *both* the measurements made by Alice and Bob's displacement. To illustrate our general strategy for improving teleportation fidelity via knowledge of the quantum alphabet we consider the following situation: Alice and Bob know that they are attempting to teleport coherent states, and they are very sure of the phase of the states, however the input amplitude is unknown. What is the best strategy Alice and Bob can take given that they know the phase of the input state and the level of squeezing? The answer is to tailor Alice's measurements and Bob's displacement to the known amount of squeezing. Bob then merely displaces his component of the entanglement resource in the known direction by an amount related to the information sent to him.

We also adapt our tailored displacement scheme to the situation where the target state alphabet is a two-dimensional distribution in phase space; agreeing with and extending previous results by Braunstein, Fuchs and Kimble [11]

Overall, one can still make use of the prior knowledge of the target state alphabet and optimise the protocol over the gain for nonzero levels of squeezing. The tailored displacement teleportation technique is again useful in improving continuous variable teleportation.

Overall, we introduce a refined measurement and displacement strategy which makes good use of the properties of prior knowledge about the target state and nonmaximal entanglement. This refinement is tailored to the given experimental situation and can give a great improvement on the output quality of continuous variable teleportation. The strategy described here is generally applicable to all teleportation schemes involving physically limited resources. A major advantage of this scheme is that it is able to be implemented with current continuous variable teleportation technology since it only requires linear gain on the measurement results.

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