Numerical analysis of a two-qubit quantum gate with a capacitive coupling

Takahiro Murakami[1] * Masataka Iinuma[1] † Tohru Takahashi[1] ‡ Yutaka Kadoya[1] § Masamichi Yamanishi[1] ¶

[1] *Graduated School of Advanced Sciences of Matter, Hiroshima University. 1-3-1 Kagamiyama, Higashi-Hiroshima, Hiroshima 739-8530, Japan.*

Abstract. Behavior of the two-qubit system coupled by the electric capacitance has been studied. It was found that the system could not be operative as an universal quantum gate unless it had capability to switch the coupling between two qubits. On the other hand, we showed that the interaction Hamiltonian for the capacitive coupling is essentially same as the dipole-dipole coupling. Therefore, the system is capable of working as the universal quantum gate by applying operation analogous to the NMR quantum computer if the coupling is weak enough. The Controlled-NOT operation has been demonstrated numerically as an example of quantum gate using weak capacitive coupling.

Keywords: two-qubit, capacitive coupling, dipole-dipole coupling, CNOT gate

For involving a function as universal quantum gates, it is well known widely that a quantum system must satisfy the following requirements, (1) the transition between two levels in each qubit which has to be controlled independently for both phase and amplitude, (2) qubits must have suitable mutual interaction to construct the CNOT gate. In a multi-qubit system allowing any unitary transformation to each single qubit independently, a typical and wide way for the mutual communication is to apply the dipole-dipole interaction, such as NMR devices. Recently, the possibility of multi-qubits designs using Josephson-junction devices with the type of the dipole-dipole coupling has been discussed in[1]. However, no discussion of a two-qubit system with charge qubits via a simple capacitive coupling has been made so far because of the explicit different type of the interaction from the dipole-dipole coupling. Recently, an observation of Rabi oscillation in a two-qubit system using the Josephson-junction device operating in the charge regime has been reported[3]. A two-qubits system using exciton-photon devices proposed by Yamanishi and Kadoya has also a similar structure of the capacitive coupling interaction[4]. However, a construction of the universal gate is yet to be demonstrated both theoretically and experimentally. In this paper, for the two-qubit system coupled by the electric capacitance, we studied the feasibility of the universal quantum gate, in which the requirements of CNOT gate operation and independent single qubit operations are fulfilled.

We analyzed the wave function of a two-qubit system in the 4-dementional vector space $\psi \equiv \varphi_1 \otimes \varphi_2$, where the basis of the space is defined explicitly as

$$\begin{pmatrix} |1> \\ |0> \end{pmatrix}_1 \otimes \begin{pmatrix} |1> \\ |0> \end{pmatrix}_2 = \begin{pmatrix} |1>|1> \\ |1>|0> \\ |0>|1> \\ |0>|0> \end{pmatrix}. \quad (1)$$

Here, we assume that a state 'bit 1' is defined as presence of an electric charge and the another state 'bit 0' is defined as absence of the electric charge. The time evolution of each qubit can be described by Schrödinger equation as $i\hbar(d\varphi_i/dt) = H_i\varphi_i$, where H_i is the Hamiltonian of i-th qubit and the explicit form is $H_i = \begin{pmatrix} \Delta_i & a_i \\ a_i & -\Delta_i \end{pmatrix}$ with Δ_i and a_i being the energy level and the Rabi oscillation strength of the qubit. In the proposed devices, it is assumed that the Rabi oscillation can be controlled by changing Δ_i through external parameters such as voltages applied to the devices. The time evolution of the wave function ψ is written as

$$\begin{aligned} i\hbar(d\psi/dt) &= H_{cap}\psi \\ H_{cap} &= H_1 \otimes I_2 + I_1 \otimes H_2 + H_{12} \end{aligned} \quad (2)$$

where H_{12} is the interaction Hamiltonian between two qubits. As a form of H_{12}, an additional energy is put to the system only when both qubits are excited. Thus the Hamiltonian H_{12} can be described as

$$H_{12} = \begin{pmatrix} \Delta_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (3)$$

where Δ_{12} is the coupling energy between two qubits. We can rewrite H_{cap} in a two-component form as

$$\begin{aligned} H'_{cap} &= \frac{\Delta_{12}}{4} I + a_1\sigma_x^1 + a_2\sigma_x^2 + \left(\Delta_1 + \frac{\Delta_{12}}{4}\right)\sigma_z^1 \\ &+ \left(\Delta_2 + \frac{\Delta_{12}}{4}\right)\sigma_z^2 + \frac{\Delta_{12}}{4}\sigma z^1\sigma_z^2, \end{aligned} \quad (4)$$

where σ_x^i,σ_z^i are x,z component of i-th Pauli-matrices, respectively, and I is the unit matrix. The coupling energy $\Delta_{12}/4$ is added to the energy of each quantum state in forth and fifth terms. The last term of the Hamiltonian [3] is the same type of the dipole-dipole coupling, which is typically used in NMR devices. This fact says that the interaction of the capacitive coupling between two charge

*murakami@qp.hiroshima-u.ac.jp
†iinuma@hiroshima-u.ac.jp
‡tohrut@hiroshima-u.ac.jp
§kd@hiroshima-u.ac.jp
¶yaman@hiroshima-u.ac.jp

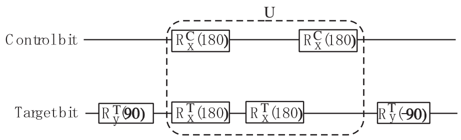

Figure 1: Block diagram of a sequence of unitary operation for construction of CNOT gate

qubits can be described essentially by the same form with the dipole-dipole couplings.

For demonstrating the universal gate with H_{cap}, we checked feasibility of both of the CNOT gate operation and the independent single qubit operation. One idea for the CNOT is that the coupling energy is adjusted as being the similar value of the energy difference between two levels of i-th qubit $2\Delta_i$, because the quantum state of i-th qubit should affect the condition of Rabi oscillation of neighboring qubit. The simplest operation can be found as following procedure. The unitary transformation at time t corresponded to H_{cap} can be expressed as $U_{st} = exp[-iH_{cap}t/\hbar]$. The parameters in U_{st} can be found as the U_{ST} becomes equivalent to the same form of the unitary transformation of the CNOT gate. Using the values of $a_1 = a_2 = a = 0.13$[meV], $2\Delta_1 = 4.87$[meV], $2\Delta_2 = 2.63$[meV], and $\Delta_{12} = 2.63$[meV], which are typically used for the exciton-photon devices except Δ_{12}[4], we computed numerically the time evolution of the phase and the probability for quantum state, $|1>|1>$, $|1>|0>$, $|0>|1>$, $|0>|0>$, respectively. In taking $\psi_{init} = (\,1\,0\,0\,0\,)$ as a initial state, a final state of $\psi_{final} = (0\,1\,0\,0\,)$ was obtained after the period of 7.85[ps] which is determined by $t = \pi/2a$. The results show that the gate is working as the CNOT gate with one unitary operation under the above parameter setting. However, the two-qubit system is not operative as the universal gate, unless a mechanism of switching the coupling ON or OFF is embedded in the hardware. Such strong coupling causes the influence of each quantum state to the Rabi oscillation of neighboring qubits and prevents the single qubit operation independently.

To realize the universal gate, the most important condition is strength of the coupling Δ_{12}. If the coupling Δ_{12} is enough smaller than the Rabi oscillation strength $a_{1(2)}$, the condition of the Rabi oscillation can be approximately independent of neighboring qubit. This situation is similar to NMR devices, which don't need switching the interaction among qubits in the hardware.

In order to see feasibility of the CNOT gate operation in the two-qubit system with more weaker coupling, we performed a numerical calculation with Eq. 2 changing the strength of the coupling. In this case, the CNOT gate operation is similar to the procedure of NMR devices. The sequence of the unitary operations is shown in Fig. 1 schematically. In the calculation, the initial state was chosen as $\psi_{init} = (\,1\,0\,0\,0\,)$ on the basis shown in Eq. 1 and the final state is expected as $\psi_{final} = (0\,e^{-i\pi/4}\,0\,0\,)$. In Fig. 2, we plotted the amplitude and the phase of the $|1>|0>$ as a function of Δ_{12} normalized to $a_{1(2)}$, where we expect 1.0 and $-\pi/4$ for the amplitude and the phase, respectively. We also calculated on the other

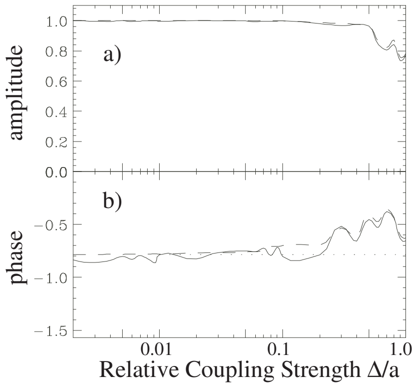

Figure 2: Phase and Amplitude as a function of relative coupling constant Δ/a

treatment of the Rabi oscillation strength $a_{1(2)}$. The parameter $a_{1(2)}$ is non-vanishing throughout the operations or appears only when the devices are working on Rabi oscillation, like NMR devices. In Fig. 2, the solid line shows the case in which the Rabi oscillation strength $a_{1(2)}$ is present throughout the whole operation or the dashed line is for the case in which $a_{1(2)}$ appears only for the Rabi oscillation stage. Fig. 2 shows that the results are reasonably close to the expected values and are stable up to $\Delta_{12} \approx 0.1a_{1(2)}$. Although it is preferable to control $a_{1(2)}$ as the NMR devices, the deviation is within an acceptable level. It is also worthwhile mentioning that $\Delta_{12} \approx 0.1a_{1(2)}$ is much larger than those of typical NMR devices. It means that the device with capacitive coupling potentially have advantage of larger signal than those of the NMR devices.

In summary, we analyzed the two-qubits system of charge qubits via a capacitive coupling. We found that the capacitive coupling is essentially the same as the dipole-dipole type interaction. Therefore, the device can be operative as the universal quantum gate if the coupling is smaller enough than the Rabi oscillation strength. In the strong coupling, we also found the unitary operation and the parameters which enable the two qubit-system to be functional as the universal gate, if it has the switching mechanism embedded in the hardware.

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