Black Hole Thermodynamics and Quantum Information Theory

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Abstract. We describe a quantum version of a gedanken experiment which supports the generalized second law of black hole thermodynamics [1]-[6]. In the black hole thermodynamics the total entropy is the sum of the black hole entropy $S_{BH} = A/4$ (where A is the area of the black hole horizon, and $S_{BH} = 4\pi M^2$ for a spherical black hole of mass M) and of the ordinary matter entropy S_M: $S_T = S_{BH} + S_M$. The quantum state of the matter in the black hole spacetime is given by the Hartle-Hawking state,

$$|\psi >_{HH} \equiv \sum_n \sqrt{c_n} |n >_B |n >_A \tag{1}$$

(where $c_n \equiv \exp[-\omega n/T_{BH}]/Z$ is the Boltzmann factor, and $T_{BH} \equiv (8\pi M)^{-1}$ is the Hawking temperature), which is an entangled state of the particles inside ($|n >_B$) and outside ($|n >_A$) of the black hole just like an EPR pair. A quantum measurement (a nonorthogonal positive operator valued measure POVM) of particles in the region outside of the event horizon decreases the entropy of the outside matter due to the entanglement of the inside and outside particle states. This decrease is compensated, however, by the increase in the detector and the black hole entropies, the latter via the increase of the black hole mass which is ultimately attributed to the work done by the measurement [7]. The main ingredients are the first law of black hole physics, $S_{BH} = \Delta M/T_{BH} = \Delta W/T_{BH}$, and the second law of ordinary thermodynamics outside the horizon, stating that the work ΔW needed for the quantum (isothermal) experiment is not smaller than the variation of the free energy, i.e. $\Delta W \geq \Delta F$. We then easily obtain the generalized second law:

$$\Delta S_T \equiv (S'_{BH} + S'_M) - (S_{BH} + S_M) \geq 0 \tag{2}$$

If the detector is further, conditionally dropped into the black hole depending on the experimental outcome ($\alpha \in D$, where α labels the possible experiment outcomes, i.e. the detector states), it can be shown that the increase of the generalized entropy by a quantum process outside the horizon of a black hole is more than the Holevo bound of the classical information which could be obtained by further observations outside the horizon [8], i.e.:

$$\Delta S'_T \geq p_D \chi_{\alpha\in D} + (1 - p_D)\chi_{\alpha\notin D}, \tag{3}$$

where $\chi_{\alpha\in D} \equiv \left[S\left(\sum_{\alpha\in D} \hat{p}_\alpha \rho'_\alpha\right) - \sum_{\alpha\in D} \hat{p}_\alpha S'_\alpha\right]$ (and similarly for $\alpha \notin D$; notation: the reduced density operator for the outside observer after the experiment and after tracing out the B and the detector states is $\rho'_A \equiv \sum_\alpha A_\alpha \rho_A A^\dagger_\alpha = \sum_\alpha p_\alpha \rho'_\alpha$, where $\rho_A \equiv \mathrm{Tr}_B(|\psi >_{HH} < \psi|)$, p_α is the probability to get the measurement result α and A_α is the detector POVM; $\hat{p}_\alpha \equiv p_\alpha/p_D$ and $\tilde{p}_\alpha \equiv p_\alpha/(1 - p_D)$ are the normalized probabilities for, respectively, $\alpha \in D$ and $\alpha \notin D$, and $p_D \equiv \sum_{\alpha\in D} p_\alpha$ is the total probability that the detector is dropped into the black hole; the entropy $S'_\alpha \equiv S(\rho'_\alpha)$) are the Holevo accessible informations which upper bound the mutual classical informations [9], $\chi'_{\alpha\in D(\alpha\notin D)} \geq I_{\alpha\in D(\alpha\notin D)}$, which would be obtained if one performed a further measurement before the detector and the outside observer get causally disconnected. The variation of the total entropy has been redefined here as $\Delta S'_T \equiv \Delta S_T + S_c$, to include the contribution of (minus) the 'entropy of the choice' [10] for the detector, $S_c \equiv -\sum_\alpha p_\alpha \log p_\alpha$, which can be interpreted as reflecting the a priori ignorance about the actual outcome of the measurement determining α. Since the amount of increase of the generalized total entropy becomes less and less at each step of measurement and eventually does not change at (due to the monotonicity of the Holevo accessible information), our results remind of Prigogine's theorem on minimum entropy production [11]. The experiment also provides an explicit realization of semilocal and acausal operations [12]. We finally suggest a possible way of going beyond the classical treatment of the black hole geometry by considering the spacetime and the (second quantized) matter degrees of freedom as a (semiclassical) solution to the Wheeler-DeWitt constraint of geometrodynamics [13]. The difficulty in defining natural 'external observers' in superspace turns the inherited entanglement between the gravitational and matter degrees of freedom (with a coherence comparable to cosmological times) to the role of a fundamental quantity.

Keywords: Quantum Information, Entanglement, Black Holes, Thermodynamics.

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