Spin-1/2 geometric phase driven by decohering quantum fields

A. Carollo[1 2 *] I. Fuentes-Guridi[3 †] M. França Santos[1 ‡] V. Vedral[1 §]

[1] *Optics Section, The Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom.*
[2] *INFM, unità di ricerca di Milano, via Celonia 16, Italy.*
[3] *Perimeter Institute, 35 King Street North Waterloo, Ontario Canada N2J 2W9.*

Abstract. We calculate the geometric phase of a spin-1/2 system driven by a one and two mode quantum field subject to decoherence. Using the quantum jump approach, we show that the corrections to the phase in the no-jump trajectory are different when considering an adiabatic and non-adiabatic evolution. We discuss the implications of our results from both the fundamental as well as quantum computational perspective.

In quantum mechanics physical states are equivalent up to a global phase which in general does not contain useful information about the described system, and thus, can be ignored. However, Berry [1] surprisingly showed that these phases can have a component of geometric origin with important observable consequences. These components which are gauge invariant and only depend on the path followed by the system during its evolution, have been investigated and tested in a variety of settings and have been generalized in several directions [2]. Geometric phases are interesting both from a fundamental point of view and for their applications, among which the geometric quantum computation [3] is one of the most important. In fact, the use of geometric phases in the implementation of fault-tolerant quantum gates has motivated their study under more realistic situations [4]. When a system interacts with the environment, the quantum superpositions decay into statistical mixtures [5] and this effect, called decoherence, is the most important limiting factor for quantum computation.

There are some works that investigate the behavior of geometric phases under some typical errors sources like random classical fluctuations to the driving fields, as well as generic reservoirs acting in spin 1/2 evolutions [6, 7]. All of them consider the driving field as a classical system. However, any driving field is also a quantized system and, in most of the typical experimental situations, this quantum behavior is relevant, especially when decoherence affects those fields. In fact, decoherence in the driving field may become critical, particularly when geometric phases are used to implement quantum protocols, like communication and computational ones.

In this work, we will present the behavior of the geometric phase of a spin 1/2 particle interacting with a driving magnetic field when this field is not only quantized but also subjected to decoherence. We calculate and analyze the effect of decoherence of the driving field on both adiabatic and non-adiabatic evolutions of the spin 1/2 particle. First we briefly describe the general framework of geometric phases in open systems, developed in [7]. Then we calculate Berry's phases for different interactions of spin 1/2 systems and decohering fields both in the adiabatic and non-adiabatic scenarios. Finally we point out the differences between these two situations and how this noise source compares to previously analyzed ones.

The approach we use to derive the geometric phase is based on a quantum-jump method recently introduced in ref. [7]. In particular we will confine ourself on the geometric phase observable in the "no-jump" trajectory [7]. Suppose that a system interacting with a reservoir, is monitored by detectors. If the detectors do not detect any decay, the geometric phase associated to system evolution is given by [7]:

$$\gamma^0 = \int_0^T \frac{\langle\psi^0(t)|H|\psi^0(t)\rangle}{\langle\psi^0(t)|\psi^0(t)\rangle}dt - \arg\{\langle\psi^0(T)|\psi^0(0)\rangle\}, \quad (1)$$

where $i\frac{d}{dt}|\psi^0(t)\rangle = \tilde{H}|\psi^0(t)\rangle$, $|\psi^0(0)\rangle = |\psi_0\rangle$, and $\tilde{H}$ is a non-Hermitian effective Hamiltonian given by $\tilde{H} = H - \frac{i}{2}\sum_{k=1}^{n}\Gamma_k^\dagger\Gamma_k$. We will apply this general method to the system considered in ref. [8]. In the above mentioned work a system composed of a two-mode quantized field and a two-level atom jointly evolving under an adiabatic transformation has been considered. Particularly, the observation of a geometric phase associated with the initial vacuum state of the field is predicted, which gives a value of $\Omega/4$, Ω being the solid angle on a parameter sphere.

We describe the two-level system with Bohr frequency ω in terms of Pauli operators σ_z, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$ and the two field modes (both with frequency ν) in terms of the creation and annihilation operators a, $a^\dagger$ and b, $b^\dagger$. In the above mentioned paper, the geometric phase is obtained by an adiabatic evolution of the initial Hamiltonian $H = \nu a^\dagger a + \nu b^\dagger b + \lambda(\sigma_+ a + \sigma_- a^\dagger)$ by means of the two-mode displacement operator $U(\phi,\alpha) = e^{-i\phi J_z}e^{-i\alpha J_y}$ where $J_z = \frac{1}{2}(a^\dagger a - b^\dagger b)$ $J_x = \frac{1}{2}(a^\dagger b + ab^\dagger)$, and $J_y = \frac{1}{2i}(a^\dagger b - ab^\dagger)$. If consider in the initial state of the evolution to be the eigenstate $|\phi\rangle = \cos\theta_n/2|e,n,n'\rangle + \sin\theta_n/2|g,n+1,n'\rangle$, after a cyclic evolution of the papameters θ and α, this state acquires a geometric phase equal to $\chi_{(n,n')} = \frac{1}{2}\Omega\left[n - n' + \frac{1}{2}(1-\cos\theta_n)\right]$, where $\Delta = \omega - \nu$ is the detuning between the quantum mode and the two-level system and λ is the coupling constant. where n and n' correspond to the number of pho-

*a.carollo@imperial.ac.uk
†ifuentes-guridi@perimeterinstitute.ca
‡m.santos@imperial.ac.uk
§v.vedral@imperial.ac.uk

tons in the a and b field modes respectively and where $\Omega = 4\pi(1-\cos\alpha)$ is the solid angle described by the parameters α and ϕ. In the special case of $\theta_n = \pi/2$ (the maximally entangle state) and $n = n' = 0$, the result: $\chi_0 = \Omega/4$ is obtained.

To consider now that the field is subject to decoherence, we assume a master equation evolution for a decaying field, which, in the no-jump case, amounts to consider an effective Hamiltonian of the form $\tilde{H} = H - i\frac{\lambda}{2}\hat{N}$ with $\hat{N} = a^\dagger a + b^\dagger b$ the total number of photons in the system. The non-Hermitiam Hamiltonian is obtained from the assumption that no jump occurs during the evolution, i.e. the system is assumed to be continuously monitored by detectors and no detection of photon is observed. In such a scenario, it can been shown that the deviation of the geometric phase from the value $\chi(n,n')$ is vanishing up to the first order in λ/R [9]. This reflects the resilience of the geometric phase against the environment.

It is interesting to compare the geometric phase due to an adiabatic evolution in presence of decoherence and the analogous result obtained in a non-adiabatic fashion. It is well known [10] that the adiabaticity is not a necessary condition to observe geometric phases. In fact, these are uniquely defined by the path on the projective Hilbert space traversed by the quantum system in its evolution. Thus, no matter how this evolution is achieved, the geometric phase will remained unchanged. This may no longer be the case in presence of decoherence, as the interaction with the environment can affects differently adiabatic and non-adiabatic evolutions.

Consider the following non-adiabatic setup: the system is initially prepared in an entangled state of atom and the field mode. Then it evolves under a time independent Hamiltonian involving only the degrees of freedom of the field. By turning on a suitable Jaynes-Cummings interaction we can prepare the system in the state $|\psi_{in}\rangle = e^{-iJ_y\alpha}|\phi\rangle$ where $|\phi\rangle = \cos\theta_n/2|e,n,n'\rangle + \sin\theta_n/2|g,n+1,n'\rangle$. After this preparation, we assume that the dynamics of the system is described, in the interaction picture, by the Hamiltonian $H_{int} = \delta J_z$, where δ is now the constant parameter. Thus, the state evolves according to $|\psi(t)\rangle = e^{-i\delta J_z t}|\psi_{in}\rangle$ and after a time $T = 4\pi/\delta$ the state completes a closed loop. Using the definition of Aharanov and Anandan geometric phase [10] it is easy to show that after a cyclic evolution the phase acquired by the state is the same as $\chi(n,n')$ with $\Omega = 4\pi(\cos\alpha)$, i.e. the solid angle spanned on the parameter sphere in the adiabatic case.

When the decoherence of the field is considered, the phase for the no-jump trajectory can be calculated from the expression (1) and it can be shown that the decoherence-free case is recovered for low values of the parameter λ. As expected the geometric phase is affected by decoherence in different ways for the adiabatic and non-adiabatic scenarios. In particular, for low decoherence rates, i.e. $\lambda << R$ and $\lambda << \beta$ in the adiabatic and non-adiabatic case, respectively, the lowest correction is quadratic in the former and linear in the latter case. This should be expected since in the adiabatic evolution the probability for the state to be dragged away by the decoherence from the unperturbed evolution is washed away (in the first order) by the driving Hamiltonian, thereby opposing against decohering effects. On the other hand, in the non-adiabatic evolution, there is no action other than the decoherence, which finds no resistance in the evolution.

Working towards having a realistic description of geometric phases we have introduced field decoherence in the problem of a two level system interaction with a quantized field. It can be shown that in the geometric phase generated by an adiabatic evolution the first correction due to the decoherence of the driving field is only of second order in the decaying rate of the field λ. This result reinforces the idea that geometric phases can be robust to decoherence effects, in agreement with previous works analyzing the geometric phase under different noise sources [6]. We also showed that, for the non-adiabatic evolution this is no longer the case, and decoherence effects appear already in the first order correction term. This result is also in accordance with previous works in which, again, different noise sources were considered [11].

Our results are particularly relevant in the experimental realizations of these phases, like the one proposed in [12], and in their use in the implementation of geometric quantum computation. Understanding the effects of decoherence in the geometric evolution of states is the first step in finding schemes resilient to this.

References

[1] Berry, M. V. *Proc. Roy. Soc. A* **392**, 45 (1984).

[2] Shapere, A. and Wilczek, F. *Geometric phases in physics.* World Scientific, Singapore, (1989).

[3] Ekert, A., et al. *J. Mod. Optic.* (47), 2501 (2000); Pachos, J., Zanardi, P., and Razetti, M. *Phys. Rev. A* **61**, 010305 (2000).

[4] Siqvist, E., et al. *Phys. Rev. Lett.* **2845**, 86 (2000); Ericsson, M., Siöqvist, E., and Brännlund, J. *e-print quant-ph 0205160* (2002).

[5] Zurek, W. H. *Phys. Rev. D* **24**, 1516 (1981); Caldeiras, A. O. and Leggett, A. J. *Phys. Rev. A* **31**, 1059 (1985).

[6] De Chiara, G. and Palma, G. M. *quant-ph/0303155* (2003);

[7] Carollo, A., Fuentes-Guridi, I., Santos, M. F., and Vedral, V. *Phys. Rev. Lett.* **90**, 160402 (2003).

[8] Fuentes-Guridi, I., Carollo, A., Bose, S., and Vedral, V. *Phys. Rev. Lett.* **89**, 220404 (2002).

[9] Carollo, A., Fuentes-Guridi, I., Santos, M. F., and Vedral, V. (2003). quant-ph /0306178.

[10] Aharonov, Y. and Anandan, J. *Phys. Rev. Lett.* **58**, 1593 (1987).

[11] Blais, A. and Tremblay, A.-M. S. *Phys. Rev. A* **67**, 012308 (2003).

[12] Carollo, A., Santos, M. F., and Vedral, V. *Phys. Rev. A* **67**, 063804 (2003).