Quantum Teleportation with Atoms Trapped in Cavities

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Abstract. We propose a scheme to implement the quantum teleportation protocol with single atoms trapped in cavities. The scheme is based on the adiabatic passage and the polarization measurement. We show that it is possible to teleport the internal state of an atom trapped in a cavity to an atom trapped in another cavity with the success probability of 1/2 and the fidelity of 1. The scheme is resistant to a number of considerable imperfections such as the violation of the Lamb-Dicke condition, weak atom-cavity coupling, spontaneous emission, and detection inefficiency.

Keywords: teleportation, cavity

In earlier proposals of quantum teleportation of single atomic states [1], qubits were internal states of single flying atoms. From the viewpoint of quantum information processing, however, it would be ideal to have atoms as stationary qubits used only for storage of information and leave communication to photons. Recent advances in cavity quantum electrodynamics techniques of trapping and manipulating atoms [2] open ways for such a scheme.

In the present work, we propose a scheme to implement quantum teleportation with single atoms each trapped in a cavity. The schematic representation of our scheme is shown in Figure. 1. The atom A is trapped in Alice's cavity, and the atom B in Bob's cavity. Each atom is driven adiabatically by a classical coherent field. The level structures of atoms will be described later. Alice maps the unknown internal state of her atom into the two-mode state of her cavity through adiabatic passage, while Bob generates a maximally entangled state of the internal state of his atom and the two-mode state of his cavity through adiabatic passage. During both the adiabatic passage processes, with the probability of 1, each cavity should emit one photon with two possible polarization degrees of freedom in which the quantum information is encoded. Two photons leaking out from both cavities interfere at the 50-50 beam splitter S. The beam splitter S, two quarter wave plates W_1 and W_2, two polarization beam splitters P_1 and P_2, and four detectors D_{1L}, D_{1R}, D_{2L}, and D_{2R} constitute a measurement device for discriminating between the Bell states of the two single-photon polarization qubits.

The involved atomic levels and transitions are depicted in Figure. 2. For the operation, both Alice and Bob exploit two $F = 1$ hyperfine levels, whereas Bob exploits one additional hyperfine level. A qubit is encoded in two Zeeman sublevels of the $F = 1$ ground hyperfine level. To express the state of the atom-cavity system, we use the notation: $|\Psi(t)\rangle_i = |x\rangle_i |n_L, n_R\rangle_i$, where $i = A, B$ denotes Alice or Bob, x the atomic state, and $n_{L,R}$ the number of left- or right-circularly polarized photons in Alice's or Bob's cavity. Transitions between the $F = 1$ ground and excited hyperfine levels, both the cavity modes, and the classical field for Alice are all resonant with the same frequency ω, whereas the transition $|g_0'\rangle_B \leftrightarrow |e_0\rangle_B$ and the classical field for Bob are resonant with another frequency ω'. $\Omega_i(t)$ and g_i represent the time-dependent Rabi frequency of the classical field (assumed to be real without loss of generality) and the atom-cavity coupling rate (assumed to be the same for both the transitions) respectively, with $i = A, B$ for Alice or Bob. For the moment, we assume that g_i remains constant during the operation. The assumption is valid in the Lamb-Dicke limit.

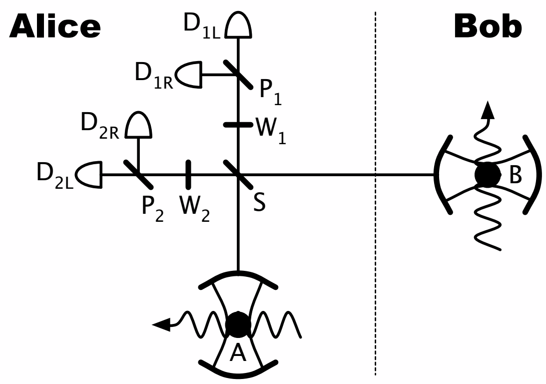

Figure 1: Experimental scheme to teleport the internal state of atom A to atom B. S is a beamsplitter, W_1 and W_2 are quarter wave plates, P_1 and P_2 are polarization beam splitters, and D_{1L}, D_{2L}, D_{1R}, and D_{2R} are photodetectors. Each winding arrow represents the classical driving field

Initially, Alice's system is prepared in the following state:

$$|\Psi(0)\rangle_A = (\alpha\,|g_L\rangle + \beta\,|g_R\rangle)_A\,|0,0\rangle_A\,, \tag{1}$$

where α and β are unknown. If the variation of $\Omega_A(t)$ is sufficiently slow, only the four transitions are involved as depicted in Fig. 2(a): $|g_m\rangle_A \to |e_m\rangle_A$ $(m = L, R)$ driven by the π-polarized classical field and $|e_L\rangle_A \to |g_0\rangle_A$ ($|e_R\rangle_A \to |g_0\rangle_A$) coupled to the left-circularly (right-circularly) polarized mode of the cavity. The transition between $|g_0\rangle_A$ and $|e_0\rangle_A$ is electric dipole forbidden. Consequently, in the rotating frame, the Hamiltonian of the total system can be written as

$$\begin{aligned} H_A &= \Omega_A(t)(|e_L\rangle\langle g_L| + |e_R\rangle\langle g_R|)_A \\ &\quad + g_A(a_L^A\,|e_L\rangle\langle g_0| + a_R^A\,|e_R\rangle\langle g_0|)_A + h.c., \end{aligned} \tag{2}$$

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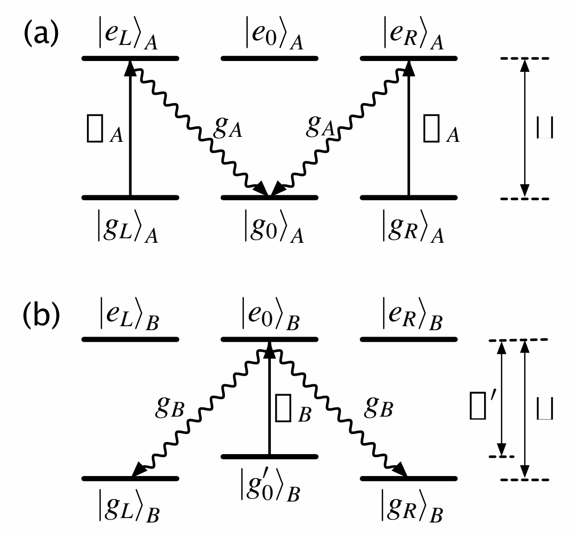

Figure 2: The involved atomic levels and transitions for Alice (a) and Bob (b). Alice's qubit is encoded in the two Zeeman sublevels $|g_L\rangle_A$ and $|g_R\rangle_A$, and Bob's qubit in the same way. Each straight arrow represents the transition driven by the π-polarized classical coherent field and each winding arrow represents the transition due to the atom-cavity coupling. Each transition of $|e_L\rangle_A \rightarrow |g_0\rangle_A$ and $|e_0\rangle_B \rightarrow |g_R\rangle_B$ ($|e_R\rangle_A \rightarrow |g_0\rangle_A$ and $|e_0\rangle_B \rightarrow |g_L\rangle_B$) is coupled to the left-circularly (right-circularly) polarized mode of the cavity. The transition $|g_0\rangle_A \leftrightarrow |e_0\rangle_A$ is electric dipole forbidden. w and w' represent the relevant transition frequencies.

where $a^A_{L,R}$ denotes the annihilation operator for the corresponding polarized mode of the cavity. The dark space is spanned by the two eigenstates $|D_1(t)\rangle_A = \cos\theta_A(t)\,|g_L\rangle_A|0,0\rangle_A - \sin\theta_A(t)\,|g_0\rangle_A|1,0\rangle_A$ and $|D_2(t)\rangle_A = \cos\theta_A(t)\,|g_R\rangle_A|0,0\rangle_A - \sin\theta_A(t)\,|g_0\rangle_A|0,1\rangle_A$, where $\theta_A(t)$ is given by $\cos\theta_A(t) = \frac{g_A}{\sqrt{|g_A|^2+|\Omega_A|^2}}$ and $\sin\theta_A(t) = \frac{\Omega_A(t)}{\sqrt{|g_A|^2+|\Omega_A|^2}}$. In the adiabatic limit, the initial state (1) evolves in the dark space into the following state:

$$\begin{aligned} |\Psi(t)\rangle_A &= \alpha\,|D_1(t)\rangle_A + \beta\,|D_2(t)\rangle_A \\ &= \cos\theta_A(t)(\alpha\,|g_L\rangle + \beta\,|g_R\rangle)_A\,|0,0\rangle_A \\ &\quad - \sin\theta_A(t)\,|g_0\rangle_A\,(\alpha\,|1,0\rangle + \beta\,|0,1\rangle)_A. \end{aligned} \quad (3)$$

Alice, thus, can map her atomic state $(\alpha\,|g_L\rangle + \beta\,|g_R\rangle)_A$ into her cavity mode state $(\alpha\,|1,0\rangle + \beta\,|0,1\rangle)_A$ by simply increasing $\sin\theta_A(t)$ adiabatically.

For Bob, the atom is initially prepared in the state $|g_0'\rangle_B\,|0,0\rangle_B$. The process for Bob is similar to that for Alice. With Ω_B varied adiabatically, only the three transitions are involved as depicted in Fig. 2(b): $|g_0'\rangle_B \rightarrow |e_0\rangle_B$ driven by the π-polarized classical field and $|e_0\rangle_B \rightarrow |g_L\rangle_B$ ($|e_0\rangle_B \rightarrow |g_R\rangle_B$) coupled to the right-circularly (left-circularly) polarized mode of the cavity. Consequently, in the rotating frame, the Hamiltonian of the total system can be written as

$$\begin{aligned} H_B &= \Omega_B(t)(|e_0\rangle\langle g_0'|)_B \\ &\quad + g_B(a^B_R\,|e_0\rangle\langle g_L| + a^B_L\,|e_0\rangle\langle g_R|)_B + h.c., \end{aligned} \quad (4)$$

where $a^B_{L,R}$ denotes the annihilation operator for the corresponding polarized mode of the cavity. In the adiabatic limit, the initial state evolves into the following dark state:

$$\begin{aligned} |\Psi(t)\rangle_B &= \cos\theta_B(t)\,|g_0'\rangle_B\,|0,0\rangle_B - \\ &\quad \sin\theta_B(t)\frac{|g_L\rangle_B\,|0,1\rangle_B + |g_R\rangle_B\,|1,0\rangle_B}{\sqrt{2}}, \end{aligned} \quad (5)$$

where $\theta_B(t)$ is given by $\cos\theta_B(t) = \frac{\sqrt{2}g_B}{\sqrt{2|g_B|^2+|\Omega_B|^2}}$ and $\sin\theta_B(t) = \frac{\Omega_B(t)}{\sqrt{2|g_B|^2+|\Omega_B|^2}}$. Bob also increase $\sin\theta_B(t)$ adiabatically to generate a maximally entangled state $\frac{(|g_L\rangle|0,1\rangle+|g_R\rangle|1,0\rangle)_B}{\sqrt{2}}$.

As $\sin\theta_A(t)$ and $\sin\theta_B(t)$ are increased, each cavity emits one photon at some instant. To illustrate the basic idea of our scheme, let us first assume that both the photons reach simultaneously at the beam splitter S. Expressing the polarizations of each photon as $|L\rangle_i$ and $|R\rangle_i$ respectively, with $i = A, B$ for Alice or Bob, the total state can be written as

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}\,|g_0\rangle_A\,(\alpha\,|L\rangle + \beta\,|R\rangle)_A(|R\rangle\,|g_L\rangle + |L\rangle\,|g_R\rangle)_B. \quad (6)$$

Now it is clear that a Bell measurement with two single-photon polarization qubits followed by the corresponding unitary operation to Bob's atom completes the quantum teleportation. As we consider only linear optical elements, the success probability of such a Bell measurement has been limited up to 1/2 [3]. Our scheme also succeeds with that probability. In our setup of Fig. 1, the Bell measurement succeeds only when the two photons are found to be oppositely polarized at two detectors. From simple calculations, it is found that when D_{1L} and D_{1R} click or D_{2L} and D_{2R} click, the state of Bob's atom collapses into the state $\alpha\,|g_L\rangle_B + \beta\,|g_R\rangle_B$, whereas when D_{1L} and D_{2R} click or D_{2L} and D_{1R} click, into the state $\alpha\,|g_L\rangle_B - \beta\,|g_R\rangle_B$. For the latter case, Bob applies an appropriate local unitary operation to his atom to transform the state into the former one.

In the actual situation, each photon leaks out from the cavity in the form of a single-photon pulse due to the random nature of the emission. In this case, $\Omega_i(t)$ should be adjusted to satisfy $\Omega_B(t) = \sqrt{2}\frac{g_B}{g_A}\Omega_A(t)$, where we have assumed that two distances between the cavities and the beam splitter are the same.

We also consider effects of various imperfections which could arise in a realistic implementation, and show that our scheme is resistant to them.

References

[1] M.H.Y. Moussa, Phys. Rev. A **55**, R3287 (1997).

[2] J. McKeever, J.R. Buck, A.D. Boozer, A. Kuzmich, H.-C. Nägerl, D.M. Stamper-Kurn, and H.J. Kimble, Phys. Rev. Lett. **90**, 133602 (2003).

[3] N. Lütkenhaus, J. Calsamiglia, and K.-A. Suominen, Phys. Rev. A **59**, 3295 (1999).